M00 P1

$$\frac{8x^3}{y}=3 \longrightarrow y=\frac{8x^3}{3}$$

$$xy-y=x^2+\frac{9}{4} \longrightarrow x\,\frac{8x^3}{3}-\frac{8x^3}{3}=x^2+\frac{9}{4}$$

$$32x^4-32x^3=12x^2+27$$

$$32x^4-32x^3-12x^2-27=0$$

| | 32 | −32 | −12 | 0 | −27 |
|---|---|---|---|---|---|
| 1'5 | | 48 | 24 | 18 | 27 |
| | 32 | 16 | 12 | 18 | 0 |

$$\boxed{x=1'5} \Rightarrow \boxed{y=9}$$

M00 P1

$$(2+3x)^{10}=\dots\dots+\binom{10}{7}2^3\cdot(3x)^7+\dots\dots=\dots\dots+\boxed{2099520\,x^7}+\dots$$

N00 P1

$$\left(x+\frac{1}{ax^2}\right)^7=\dots\dots+\binom{7}{2}x^5\left(\frac{1}{ax^2}\right)^2+\dots\dots=\dots\dots+\frac{21}{a^2}x+\dots$$

$$\frac{21}{a^2}=\frac{7}{3} \Rightarrow \boxed{a=\pm 3}$$

M01 P1

$Kx^2-3x+(K+2)=0$ , puede ser una ecuación de 1.^er^ o 2º grado.

- Si $K=0$ la ecuación sería de 1.^er^ grado con solución real: $x=\frac{2}{3}$
- Si $K\neq 0$ sería de 2º grado, con soluciones: $x=\frac{3\pm\sqrt{9-4K(K+2)}}{2K}=\frac{3\pm\sqrt{9-4K^2-8K}}{2K}$

Dos raíces distintas $\Rightarrow 9-4K^2-8K>0$ ; $4K^2+8K-9<0$

$4K^2+8K-9<0$

$4K^2+8K-9=0$ ; $K=\frac{-8\pm\sqrt{64+144}}{8}=\frac{-8\pm\sqrt{208}}{8}=\frac{-8\pm4\sqrt{13}}{8}=\frac{-2\pm\sqrt{13}}{2}=-1\pm\frac{\sqrt{13}}{2}$

| Signo $4K^2+8K-9$ | | $-1-\frac{\sqrt{13}}{2}$ | | $-1+\frac{\sqrt{13}}{2}$ | |
|---|---|---|---|---|---|
| | + | | − | | + |

$$\Rightarrow \boxed{K\in\left(-1-\frac{\sqrt{13}}{2},\,-1+\frac{\sqrt{13}}{2}\right)-\{0\}}$$

N01 P1

$|5-3x|\leq|x+1|$

$|5-3x|=|x+1|$

| | |
|---|---|
| $5-3x=x+1$ | $5-3x=-(x+1)$ |
| $-4x=-4$ | $5-3x=-x-1$ |
| $x=1$ | $-2x=-6$ |
| | $x=3$ |

| ¿ $\lvert 5-3x\rvert\leq\lvert x+1\rvert$ ? | | 1 | | 3 | |
|---|---|---|---|---|---|
| | NO | | SÍ | | NO |

$$\Rightarrow \boxed{x\in[1,3]}$$

N01 P1

$f(2)=f(-1) \Rightarrow 8+12+2a+\cancel{b}=-1+3-a+\cancel{b}$ ; $3a=-18$ ; $\boxed{a=-6}$

M02 P1

$(e^x-2)(e^x-3)\leq 2e^x$

$t=e^x \longrightarrow (t-2)(t-3)\leq 2t$

$t^2-3t-2t+6\leq 2t$

$t^2-7t+6\leq 0$

$t^2-7t+6=0 \longrightarrow t=\frac{7\pm\sqrt{49-24}}{2}=\frac{7\pm5}{2}\begin{cases}6\\1\end{cases}$

| Signo $t^2-7t+6$ | | 1 | | 6 | |
|---|---|---|---|---|---|
| | + | | − | | + |

$\Downarrow$

$t\in[1,6]$

$e^x\in[1,6] \Rightarrow \boxed{x\in[0,\ln 6]}$

N02 P1

$x^2-4+\frac{3}{x}<0$

$x^2-4+\frac{3}{x}=0$ ; $x^3-4x+3=0$

| | 1 | 0 | −4 | 3 |
|---|---|---|---|---|
| 1 | | 1 | 1 | −3 |
| | 1 | 1 | −3 | 0 |

$\rightarrow x=1$

$x^2+x-3=0$

$x=\frac{-1\pm\sqrt{1+12}}{2}=\frac{-1\pm\sqrt{13}}{2}$

Signo $x^2-4+\frac{3}{x}$

| | $\frac{-1-\sqrt{13}}{2}$ | 0 | 1 | $\frac{-1+\sqrt{13}}{2}$ |
|---|---|---|---|---|
| + | − | + | − | + |

$\boxed{x\in\left(\frac{-1-\sqrt{13}}{2},0\right)\cup\left(1,\frac{-1+\sqrt{13}}{2}\right)}$

N02 P1

$\left(1-\frac{1}{2}x\right)^8=\ldots\ldots-\binom{8}{3}\cdot1^5\cdot\left(\frac{1}{2}x\right)^3+\ldots=\ldots\boxed{-7x^3}\ldots$

N02 P1

$1^4+a\cdot1+3=8 \Rightarrow \boxed{a=4}$

M03 P1

$|x-2|\geq|2x+1|$

$|x-2|=|2x+1|$

$x-2=2x+1$ ; $-x=3$ ; $x=-3$

$x-2=-(2x+1)$ ; $x-2=-2x-1$ ; $3x=1$ ; $x=1/3$

¿ $|x-2|\geq|2x+1|$ ?

| | −3 | | 1/3 |
|---|---|---|---|
| NO | | SÍ | NO |

$\boxed{x\in[-3,1/3]}$

M03 P1

$\left.\begin{array}{l}8+4a-6+b=0\\-1+a+3+b=6\end{array}\right\} \rightarrow \begin{array}{l}4a+b=-2\\-\ a+b=4\\\hline 3a\quad=-6\end{array}$

$\boxed{a=-2} \rightarrow \boxed{b=6}$

N03 P1

$(1+2k)x^2-10x+k-2=0$ , puede ser una ecuación de 1$^{er}$ o 2° grado.

- Si $1+2k\neq0$, sería de 2° grado, con soluciones: $x=\frac{10\pm\sqrt{100-4(1+2k)(k-2)}}{2(1+2k)}$

Soluciones Reales $\Rightarrow$ $100-4(1+2k)(k-2)\geq0$

$100-4(k-2+2k^2-4k)\geq0$

$100-4k+8-8k^2+16k\geq0$

$8k^2-12k-108\leq0$

$8k^2-12k-108=0$

$k=\frac{12\pm\sqrt{144+3456}}{16}=\frac{12\pm60}{16}$ → $9/2$ ; $-3$

Signo $8k^2-12k-108$

| | −3 | | 9/2 |
|---|---|---|---|
| + | | − | + |

$\boxed{k\in\left[-3,\frac{9}{2}\right]}$

- Si $k=-\frac{1}{2}$ la ecuación sería de 1$^{er}$ grado con solución real:

$0\cdot x^2-10x+\frac{-1}{2}-2=0$ ; $-10x=\frac{5}{2} \Rightarrow x=-\frac{1}{5}$ , pero $k=-\frac{1}{2}\in\left[-3,\frac{9}{2}\right]$, solución final.

M04
P1

$$\left|\frac{x+12}{x-12}\right| \leq 3$$

$$\left|\frac{x+12}{x-12}\right| = 3 \quad ; \quad |x+12| = 3|x-12|$$

$$\begin{array}{l|l} x+12 = 3(x-12) & x+12 = -3(x-12) \\ x+12 = 3x-36 & x+12 = -3x+36 \\ -2x = -48 & 4x = 24 \\ x = 24 & x = 6 \end{array}$$

¿ $\left|\frac{x+12}{x-12}\right| \leq 3$ ?

| | 6 | 12 | 24 | |
|---|---|---|---|---|
| SÍ | | NO | NO | SÍ |

$\Rightarrow \boxed{x \in (-\infty, 6] \cup [24, +\infty)}$

M04
P1

$$\left.\begin{array}{l} 1-2+a+b=0 \\ -1-2-a+b=8 \end{array}\right\} \rightarrow \left.\begin{array}{l} a+b=1 \\ -a+b=11 \end{array}\right\}$$

$$2b = 12$$

$$\boxed{b=6} \rightarrow \boxed{a=-5}$$

M04
P1

$$\left|\frac{x+9}{x-9}\right| \leq 2$$

$$\left|\frac{x+9}{x-9}\right| = 2 \quad ; \quad |x+9| = 2|x-9|$$

$$\begin{array}{l|l} x+9 = 2(x-9) & x+9 = -2(x-9) \\ x+9 = 2x-18 & x+9 = -2x+18 \\ -x = -27 & 3x = 9 \\ x = 27 & x = 3 \end{array}$$

¿ $\left|\frac{x+9}{x-9}\right| \leq 2$ ?

| | 3 | 9 | 27 | |
|---|---|---|---|---|
| SÍ | | NO | NO | SÍ |

$\Rightarrow \boxed{x \in (-\infty, 3] \cup [27, +\infty)}$

M04
P1

$$\begin{array}{rrrr|l} x^3 & + (a-4)x^2 & + (3-4a)x & + 3 & x^2-4x+3 \\ -x^3 & + 4x^2 & - 3x & & x+a \\ \hline & ax^2 & - 4ax & + 3 & \\ & -ax^2 & + 4ax & - 3a & \\ \hline & & & 3-3a & \end{array}$$

$3-3a = 0 \Rightarrow \boxed{a=1}$

N04
P1

$m(x+1) \leq x^2$

$0 \leq x^2 - mx - m$ Para Todo $x$

$x^2 - mx - m = 0$ No Tendrá soluciones reales.

$$x = \frac{m \pm \sqrt{m^2+4m}}{2}$$

$m^2 + 4m \leq 0$

$m(m+4) = 0 \rightarrow m = 0, \ -4$

Signo $m^2+4m$

| | -4 | | 0 | |
|---|---|---|---|---|
| + | | − | | + |

$\Rightarrow \boxed{m \in [-4, 0]}$

N04
P1

$(x+2)^5 = x^5 + 5x^4 \cdot 2 + 10x^3 \cdot 2^2 + 10x^2 \cdot 2^3 + 5x \cdot 2^4 + 2^5 = \boxed{x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32}$

$2'01^5 = (0'01+2)^5 = (10^{-2}+2)^5 = (10^{-2})^5 + 10 \cdot (10^{-2})^4 + 40 (10^{-2})^3 + 80 \cdot (10^{-2})^2 + 80 \cdot 10^{-2} + 32 =$

$= 10^{-10} + 10^{-7} + 4 \cdot 10^{-5} + 8 \cdot 10^{-3} + 8 \cdot 10^{-1} + 32 = \boxed{32'8080401001}$

N04
P1

$f(-2)=0 \longrightarrow (-2)^3-2\cdot(-2)^2-5\cdot(-2)+K=0 \; ; \; -8-8+10+K=0 \; ; \; \boxed{K=6}$

M05
P1

$\left(3x-\frac{2}{x}\right)^5 = \cdots + \binom{5}{2}(3x)^3\cdot\left(-\frac{2}{x}\right)^2 + \cdots = \cdots + 10\cdot 27x^3\cdot\frac{4}{x^2} + \cdots = \cdots \boxed{+1080x} \cdots$

N05
P1

$\left.\begin{array}{l} P(1)=-2 \longrightarrow 4+p+q+1=-2 \\ P(1/2)=\frac{13}{4} \longrightarrow 4\cdot\frac{1}{8}+p\cdot\frac{1}{4}+q\cdot\frac{1}{2}+1=\frac{13}{4} \end{array}\right\} ; \quad \left.\begin{array}{l} p+q=-7 \\ 4+2p+4q+8=26 \end{array}\right\} ; \quad \left.\begin{array}{l} p+q=-7 \\ 2p+4q=14 \end{array}\right\} ;$

$\left.\begin{array}{l} p+q=-7 \\ p+2q=7 \end{array}\right\}$

$-q=-14 \Rightarrow \boxed{q=14} \longrightarrow \boxed{p=-21}$

M06
P1

$\left.\begin{array}{l} P(1)=0 \longrightarrow 2+a-4+b=0 \\ P(-3)=0 \longrightarrow -54+9a+12+b=0 \end{array}\right\} \quad \left.\begin{array}{l} a+b=2 \\ 9a+b=42 \end{array}\right\}$

$8a=40 \longrightarrow \boxed{a=5} \quad \boxed{b=-3}$

N06
P1

$(\sqrt{3}-2)^3 = (\sqrt{3})^3 - 3\cdot(\sqrt{3})^2\cdot 2 + 3\sqrt{3}\cdot 2^2 - 2^3 = 3\sqrt{3}-18+12\sqrt{3}-8 = \boxed{15\sqrt{3}-26}$

M07
P1

$Kx^2-(K-3)x+(K-8)=0$ Puede ser una ecuación de 1° o 2° grado.

- Si $K\neq 0$ es una ecuación de 2° grado, con soluciones: $x=\frac{K-3\pm\sqrt{(K-3)^2-4K(K-8)}}{2K}$

Si no tiene soluciones $\Rightarrow$ $(K-3)^2-4K(K-8)<0$

$K^2-6K+9-4K^2+32K<0$

$-3K^2+26K+9<0$

$3K^2-26K-9>0$

$3K^2-26K-9=0$

$K=\frac{26\pm\sqrt{676+108}}{6}=\frac{26\pm\sqrt{784}}{6}\begin{cases}9\\-1/3\end{cases}$

Signo $3K^2-26K-9$

| | $-\frac{1}{3}$ | | 9 | |
|---|---|---|---|---|
| + | | − | | + |

- Si $K=0$, Tendría una solución real, pero vemos que este valor está excluido del intervalo: $\boxed{K\in\left(-\infty,-\frac{1}{3}\right)\cup(9,+\infty)}$

M07
P1

$f(x)=x^2-2x+K(3K+2)$

$x^2-2x+K(3K+2)=0$

$x=\frac{2\pm\sqrt{4-4K(3K+2)}}{2}$

Raíces reales distintas $\Rightarrow$ $4-4K(3K+2)>0$ ; $4-12K^2-8K>0$

$12K^2+8K-4<0$

$3K^2+2K-1<0$

$3K^2+2K-1=0$

$K=\frac{-2\pm\sqrt{4+12}}{6}=\frac{-2\pm 4}{6}\begin{cases}1/3\\-1\end{cases}$

Signo $3K^2+2K-1$

| | $-1$ | | $1/3$ | |
|---|---|---|---|---|
| + | | − | | + |

$\boxed{K\in(-1,\,1/3)}$

N07
P1

$\left.\begin{array}{l} f(2)=0 \Rightarrow 2^3+p\cdot 2^2+q\cdot 2+4=0 \\ f(-2)=0 \Rightarrow (-2)^3+p\cdot(-2)^2+q\cdot(-2)+4=0 \end{array}\right\} \quad \left.\begin{array}{l} 4p+2q=-12 \\ 4p-2q=4 \end{array}\right\}$

$8p=-8 \Rightarrow \boxed{p=-1} \quad \boxed{q=4}$

**Muestra 08 P1**

$$p^3+p^2-5p-2$$

$$p=2 \rightarrow 2^3+2^2-5\cdot 2-2 = 8+4-10-2=0 \checkmark$$

$$p^3+p^2-5p-2 = (p-2)(p^2+ap+b) = p^3+ap^2+bp-2p^2-2ap-2b =$$

$$= p^3+(a-2)p^2+(b-2a)p-2b$$

$$\left.\begin{array}{l} 1=a-2 \rightarrow \boxed{a=3} \\ -5=b-2a \longrightarrow -5=1-2\cdot 3 \;;\; -5=-5 \checkmark \\ -2=-2b \rightarrow \boxed{b=1} \end{array}\right.$$

$$p^3+p^2-5p-2 = (p-2)(p^2+3p+1)$$

$$p^2+3p+1=0 \;;\; p=\frac{-3\pm\sqrt{9-4}}{2} = \boxed{\frac{-3\pm\sqrt{5}}{2}}$$

**Muestra 08 P1**

$$\frac{2x}{|x-1|}<1$$

$$\frac{2x}{|x-1|}=1 \;;\; 2x=|x-1|$$

$$\left.\begin{array}{l} 2x=x-1 \\ x=-1 \end{array}\right\} \quad \begin{array}{l} -2x=x-1 \\ -3x=-1 \\ x=1/3 \end{array}$$

$\dfrac{2x}{|x-1|}<1$?

| | $-1$ | $1/3$ | $1$ | |
|---|---|---|---|---|
| sí | | sí | NO | NO |

$$\boxed{x\in(-\infty,1/3)}$$

**M08 P2**

$$(1-2x)^5 = 1-5\cdot(2x)+10\cdot(2x)^2-\cdots = 1-10x+40x^2\cdots$$

$$(1+x)^7 = 1+7x+21x^2+\cdots$$

$$(1-2x)^5(1+x)^7 = 1-10x+40x^2+7x-70x^2+21x^2+\cdots =$$

$$= \boxed{1-3x-9x^2}+\cdots$$

Grado 0 : 0+0

Grado 1 : $\left\{\begin{array}{l}0+1\\1+0\end{array}\right.$

Grado 2 : $\left\{\begin{array}{l}0+2\\1+1\\2+0\end{array}\right.$

**M08 P1**

$$\left.\begin{array}{l} P(-1)=0 \rightarrow -1+a-b+2=0 \\ P(2)=0 \rightarrow 8+4a+2b+2=0 \end{array}\right\} \quad \left.\begin{array}{l} a-b=-1 \\ 4a+2b=-10 \end{array}\right\} \quad \left.\begin{array}{l} a-b=-1 \\ 2a+b=-5 \end{array}\right\}$$

$$3a=-6 \;;\; \boxed{a=-2} \rightarrow \boxed{b=-1}$$

**N08 P1**

$$\left.\begin{array}{l} f(2)=15 \rightarrow 16+24+4p-4+q=15 \\ f(-3)=0 \rightarrow \cancel{81}-\cancel{81}+9p+6+q=0 \end{array}\right\}$$

$$\left.\begin{array}{l} 4p+q=-21 \\ 9p+q=-6 \end{array}\right\}$$

$$5p=15 \Rightarrow \boxed{p=3} \rightarrow \boxed{q=-33}$$

**M09 P1**

$$q(-1)=7\cdot q(-2) \rightarrow -1+k+7+3 = 7\cdot(-8+4k+14+3)$$

$$k+9=7(4k+9)$$

$$k+9=28k+63 \;;\; -27k=54 \;;\; \boxed{k=-2}$$

M09 P1

a) $V = a^3 - \left(\frac{1}{a}\right)^3 = a^3 - 3a^2\cdot\frac{1}{a} + 3a^2\cdot\frac{1}{a} + 3a\cdot\left(\frac{1}{a}\right)^2 - 3a\cdot\left(\frac{1}{a}\right)^2 - \left(\frac{1}{a}\right)^3 =$

$= \left(a-\frac{1}{a}\right)^3 + 3a^2\cdot\frac{1}{a} - 3a\cdot\left(\frac{1}{a}\right)^2 = \left(a-\frac{1}{a}\right)^3 + 3a - \frac{3}{a} =$

$= \left(a-\frac{1}{a}\right)^3 + 3\left(a-\frac{1}{a}\right) \Rightarrow \boxed{V = x^3+3x}$

b) $V = 4x \Rightarrow x^3+3x = 4x$ ; $x^3 - x = 0$ ; $x(x^2-1)=0 \begin{cases} x=0 \\ x=\pm 1 \end{cases}$

$x=0 \Rightarrow a-\frac{1}{a}=0$ ; $a=\frac{1}{a}$ ; $a^2=1$ ; $a = \begin{cases} \cancel{1} & \text{porque dicen que } a>1 \\ \cancel{-1} & \text{porque " " "} \end{cases}$

$x=1 \Rightarrow a-\frac{1}{a}=1$ ; $a^2-1=a$ ; $a^2-a-1=0$ ; $a=\frac{1\pm\sqrt{1+4}}{2} = \begin{cases} \boxed{\frac{1+\sqrt{5}}{2}} \\ \cancel{\frac{1-\sqrt{5}}{2}} & \text{porque } a>1 \end{cases}$

$x=-1 \Rightarrow a-\frac{1}{a}=-1$ ; $a^2-1=-a$ ; $a^2+a-1=0$ ; $a=\frac{-1\pm\sqrt{5}}{2} \begin{cases} \cancel{\frac{-1+\sqrt{5}}{2}} & \text{"} \\ \cancel{\frac{-1-\sqrt{5}}{2}} & \text{"} \end{cases}$

N09 P2

a) $\left(1+\frac{x}{2}\right)^m = \cdots + \binom{m}{3}\cdot 1^{m-3}\cdot\left(\frac{x}{2}\right)^3 + \cdots$

$\frac{m!}{(m-3)!\cdot 3!}\cdot 1\cdot\frac{\cancel{x^3}}{8} = 70\cancel{x^3}$

$\frac{m(m-1)(m-2)}{48} = 70$ ; $m(m^2-3m+2) = 3360$
$m^3-3m^2+2m-3360=0$

$$\begin{array}{r|rrrr} & 1 & -3 & 2 & -3360 \\ 16 & & 16 & 208 & 3360 \\ \hline & 1 & 13 & 210 & \boxed{0} \end{array} \Rightarrow \boxed{m=16}$$

b) $\left(1+\frac{x}{2}\right)^{16} = \cdots + \binom{16}{2}\cdot 1^{14}\cdot\left(\frac{x}{2}\right)^2 + \cdots =$

$= \cdots + 120\cdot\frac{x^2}{4} + \cdots = \cdots + \boxed{30x^2} + \cdots$

N09 P1

a) Si $x=-1'796$ es raíz de $x^3-x+4=0$ significa que $(-1'796)^3-(-1'796)+4=0$ por lo tanto $\boxed{x=-0'796}$ es raíz de $(x-1)^3-(x-1)+4=0$, ya que al sustituir resulta: $(-0'796-1)^3-(-0'796-1)+4 = (-1'796)^3-(-1'796)+4=0$ ✓

b) $8x^3-2x+4=0 \longrightarrow (2x)^3-(2x)+4=0$

Por lo tanto $x=\frac{-1'796}{2}=\boxed{-0'898}$ será raíz de $8x^3-2x+4=0$, ya que al sustituir resulta: $[2\cdot(-0'898)]^3-[2\cdot(-0'898)]+4 = (-1'796)^3-(-1'796)+4=0$ ✓

N09 P1

$3\cdot 1^5 - a\cdot 1 + b = 3\cdot(-1)^5 - a\cdot(-1)+b$ ; $3-a+\cancel{b} = -3+a+\cancel{b}$ ; $-2a=-6$ ; $\boxed{a=3}$

Como 'b' queda simplificado, puede tomar cualquier valor: $\boxed{b\in\mathbb{R}}$

M10 P1

Si $Ax^3+Bx^2+x+6$ es divisible entre $(x+1)\cdot(x-2)$ es que es divisible por ambos, por lo tanto:

$\left.\begin{array}{l} A\cdot(-1)^3+B\cdot(-1)^2+(-1)+6=0 \\ A\cdot 2^3+B\cdot 2^2+2+6=0 \end{array}\right\}$ : $\left.\begin{array}{l} -A+B=-5 \\ 8A+4B=-8 \end{array}\right\} \longrightarrow \left.\begin{array}{l} -8A+8B=-40 \\ 8A+4B=-8 \end{array}\right\}$

$12B=-48$

$\boxed{B=-4} \Rightarrow \boxed{A=1}$

M10 P2

$$\binom{m}{3}-\binom{2m}{2}=\frac{m!}{3!\,(m-3)!}-\frac{(2m)!}{2!\,(2m-2)!}=\frac{m(m-1)(m-2)}{6}-\frac{(2m)(2m-1)}{2}=$$

$$=\frac{m(m-1)(m-2)-6m(2m-1)}{6}=\frac{m\cdot\left((m-1)(m-2)-6(2m-1)\right)}{6}=$$

$$=\frac{m\,(m^2-2m-m+2-12m+6)}{6}=\boxed{\frac{m\,(m^2-15m+8)}{6}}$$

$$\binom{m}{3}-\binom{2m}{2}>32m \Rightarrow \frac{m\,(m^2-15m+8)}{6}>32m \;;\; m^3-15m^2+8m>192m\;;$$

$$m^3-15m^2-184m>0$$

$$m^3-15m^2-184m=0$$

$$m\,(m^2-15m-184)=0 \to \cancel{m=0} \quad \text{Dicen que } m>3$$

$$\to m^2-15m-184=0\;;\quad m=\frac{15\pm\sqrt{225+736}}{2}=\frac{15\pm31}{2}\quad 23,\ \cancel{-8}\ \text{dicen que } m>3$$

Signo $m^3-15m^2-184m$

| 3 | 23 | $+\infty$ |
|---|---|---|
| $-$ | $+$ | |

$\Rightarrow \boxed{m>23} \Rightarrow \boxed{m\geq 24}$

N10 P1

$$\left(x^2-\frac{2}{x}\right)^4=(x^2)^4-4\cdot(x^2)^3\cdot\frac{2}{x}+6\,(x^2)^2\cdot\left(\frac{2}{x}\right)^2-4\cdot x^2\cdot\left(\frac{2}{x}\right)^3+\left(\frac{2}{x}\right)^4=$$

$$=\boxed{x^8-8x^5+24x^2-\frac{32}{x}+\frac{16}{x^4}}$$

N10 P1

$$|x-1|>|2x-1|$$

$$|x-1|=|2x-1|$$

$$x-1=2x-1 \quad -x=0 \quad x=0$$

$$x-1=-(2x-1) \quad x-1=-2x+1 \quad 3x=2 \quad x=2/3$$

¿$|x-1|>|2x-1|$?

| | 0 | 2/3 |
|---|---|---|
| No | Sí | No |

$\boxed{x\in\left(0,\frac{2}{3}\right)}$

M11 P2

a)

$$2x^2+x-3=0$$

$$x=\frac{-1\pm\sqrt{1+24}}{4}=\frac{-1\pm5}{4}=\begin{cases}1\\-3/2\end{cases}\Rightarrow \boxed{2x^2+x-3=2(x-1)\left(x+\frac{3}{2}\right)}$$

También:

$$\begin{array}{c|ccc} & 2 & 1 & -3\\ 1 & & 2 & 3\\ \hline & 2 & 3 & 0\end{array}\Rightarrow \boxed{2x^2+x-3=(x-1)(2x+3)}$$

b)

$$(2x^2+x-3)^8=(x-1)^8(2x+3)^8$$

$$(x-1)^8=x^8-\cdots\cdots-8x+1$$

$$(2x+3)^8=(2x)^8+\cdots\cdots+8\cdot(2x)\cdot3^7+3^8=$$

$$=256x^8+\cdots\cdots+34992x+6561$$

Coeficiente de $x$ en $(x-1)^8\cdot(2x+3)^8$ : $(1\cdot34992-8\cdot6561)x=\boxed{-17496x}$

M11 P2 a) $f(a)=-10 \to 4a^3+2a^2-7a=-10\;;\quad 4a^3+2a^2-7a+10=0$

$$\begin{array}{c|cccc} & 4 & 2 & -7 & 10\\ -2 & & -8 & 12 & -10\\ \hline & 4 & -6 & 5 & 0\end{array}\Rightarrow \boxed{a=-2}$$

M12 TZ2 P1#1

$$2\cdot(-1)^3+K\cdot(-1)^2+6(-1)+32=(-1)^4-6(-1)^2-K^2(-1)+9$$
$$-2+K-\cancel{6}+32=1-\cancel{6}+K^2+9$$
$$0=K^2-K-20$$
$$K=\frac{1\pm\sqrt{1+80}}{2}=\frac{1\pm 9}{2}\begin{cases}5\\-4\end{cases}$$

M12 TZ2 P1#4

a) $$\left(x-\frac{2}{x}\right)^4=\sum_{m=0}^{4}\binom{4}{m}x^{4-m}\cdot\left(-\frac{2}{x}\right)^m=$$
$$=x^4+4x^3\left(-\frac{2}{x}\right)^1+6x^2\left(-\frac{2}{x}\right)^2+4x\left(\frac{-2}{x}\right)^3+\left(\frac{-2}{x}\right)^4=$$
$$=\boxed{x^4-8x^2+24-\frac{32}{x^2}+\frac{16}{x^4}}$$

b) $$(2x^2+1)\cdot\left(x-\frac{2}{x}\right)^2=(2x^2+1)\left(x^4-8x^2+24-\frac{32}{x^2}+\frac{16}{x^4}\right)$$

Término constante $=2x^2\cdot\frac{-32}{x^2}+1\cdot 24=-64+24=\boxed{-40}$

M12 TZ1 P2#9

$$\left(x-\frac{2}{x}\right)^4=\cdots+\binom{4}{2}x^2\cdot\left(\frac{-2}{x}\right)^2+\cdots=\cdots+6x^2\cdot\frac{4}{x^2}+\cdots=\cdots+24+\cdots$$
$$\left(x^2+\frac{2}{x}\right)^3=\cdots+\binom{3}{2}(x^2)^1\cdot\left(\frac{2}{x}\right)^2+\cdots=\cdots+3x^2\cdot\frac{4}{x^2}+\cdots=\cdots+12+\cdots$$

Término constante $=24\times 12=\boxed{288}$

No habría más posibilidades porque las potencias de $x$ en el primer desarrollo son: $x^4\ x^2\ x^0\ x^{-2}\ x^{-4}$ y los del segundo desarrollo: $x^6\ x^3\ x^0\ x^{-3}$.

También:
$$\left(x-\frac{2}{x}\right)^4=\sum_{r=0}^{4}\binom{4}{r}x^{4-r}\left(-\frac{2}{x}\right)^r=\sum_{r=0}^{4}\binom{4}{r}(-2)^r\cdot x^{4-2r}$$
$$\left(x^2+\frac{2}{x}\right)^3=\sum_{s=0}^{3}\binom{3}{s}\cdot(x^2)^{3-s}\cdot\left(\frac{2}{x}\right)^s=\sum_{s=0}^{3}\binom{3}{s}2^s\cdot\frac{x^{6-2s}}{x^s}=\sum_{s=0}^{3}2^s\cdot x^{6-3s}$$
$$\left(x-\frac{2}{x}\right)^4\left(x^2+\frac{2}{x}\right)^3=\sum_{r=0}^{4}\sum_{s=0}^{3}\binom{4}{r}\cdot\binom{3}{s}(-2)^r\cdot 2^s\cdot x^{4-2r}\cdot x^{6-3s}$$
$$=\sum_{r=0}^{4}\sum_{s=0}^{3}\binom{4}{r}\binom{3}{s}(-1)^r\cdot 2^{r+s}\cdot x^{10-2r-3s}$$

Para ser constante: $10-2r-3s=0$

$$\underbrace{10-2r}_{\dot{2}}=3s \Rightarrow 3s=\dot{2}\Rightarrow\boxed{s=2}\to\boxed{r=2}$$

que son los sumandos constantes.

N12 P1#2

$$\left(\frac{x}{y}-\frac{y}{x}\right)^4=\sum_{r=0}^{4}\binom{4}{r}\left(\frac{x}{y}\right)^{4-r}\cdot\left(-\frac{y}{x}\right)^r=$$
$$=\sum_{r=0}^{4}\binom{4}{r}\frac{x^{4-r}}{y^{4-r}}\cdot(-1)^r\frac{y^r}{x^r}=\sum_{r=0}^{4}\binom{4}{r}(-1)^r x^{4-2r}\cdot y^{2r-4}=$$
$$=x^4\cdot y^{-4}-4x^2y^{-2}+6-4x^{-2}\cdot y^2+x^{-4}\cdot y^4=$$
$$=\boxed{\left(\frac{x}{y}\right)^4-4\left(\frac{x}{y}\right)^2+6-4\left(\frac{y}{x}\right)^2+\left(\frac{y}{x}\right)^4}=\boxed{\frac{x^8-4x^6y^2+6x^4y^4-4x^2y^6+y^8}{x^4y^4}}$$

N12
P2#2

$$x^2-(5-k)x-(k+2)=0$$

$$x=\frac{5-k\pm\sqrt{(5-k)^2+4(k+2)}}{2}=\frac{5-k\pm\sqrt{25-10k+k^2+4k+8}}{2}=\frac{5-k\pm\sqrt{k^2-6k+33}}{2}$$

$$k^2-6k+33=0$$
$$k=\frac{6\pm\sqrt{36-132}}{2}$$

$-\infty$ $\infty$
+

$k^2-6k+33$ no se anula para ningún $k\in\mathbb{R}$ siendo siempre estrictamente positivo, por lo que la ecuación de 2º grado tendrá dos soluciones reales y distintas para todo $k\in\mathbb{R}$.

M13
TZ2
P1#3

$$(2-3x)^5=2^5-5\cdot2^4(3x)+10\cdot2^3\cdot(3x)^2-10\cdot2^2\cdot(3x)^3+5\cdot2\cdot(3x)^4-(3x)^5=$$
$$=\boxed{32-240x+720x^2-1080x^3+810x^4-243x^5}$$

M13
TZ1
P2#9

a) $3x^2+2kx+k-1=0$

$$x=\frac{-2k\pm\sqrt{4k^2-12(k-1)}}{6}=\frac{-2k\pm\sqrt{4k^2-12k+12}}{6}$$

$4k^2-12k+12=0$ ; $k^2-3k+3=0$
$$k=\frac{3\pm\sqrt{9-12}}{2}$$

$-\infty$ $+\infty$
+

$4k^2-12k+12$ no se anula para ningún $k\in\mathbb{R}$ siendo siempre estrictamente mayor que cero, por lo que la ecuación tendrá dos soluciones reales y distintas para todo $k\in\mathbb{R}$

b) $$x_1=\frac{-2k+\sqrt{4k^2-12k+12}}{6}$$
$$x_2=\frac{-2k-\sqrt{4k^2-12k+12}}{6}$$
$$\Rightarrow x_1-x_2=\frac{2\sqrt{4k^2-12k+12}}{6}=\frac{\sqrt{4k^2-12k+12}}{3}$$

$4k^2-12k+12$ tomará el menor valor en su vértice $\Rightarrow k=\frac{12}{8}=\boxed{\frac{3}{2}}$

N13
P1#1

$$\left.\begin{array}{l}3\cdot(-2)^3+p(-2)^2+q\cdot(-2)-2=0\\3\cdot(-1)^3+p\cdot(-1)^2+q\cdot(-1)-2=4\end{array}\right\}\ ;\ \left.\begin{array}{l}-24+4p-2q-2=0\\-3+p-q-2=4\end{array}\right\}\ ;\ \left.\begin{array}{l}4p-2q=26\\p-q=9\end{array}\right\}$$

$$\left.\begin{array}{l}2p-q=13\\-\ p-q=9\end{array}\right\}$$
$$\boxed{p=4}\rightarrow\boxed{q=-5}$$

Muestra 14 P1 #2

a) $9x^3 - 45x^2 + 74x - 40 = 0$

$$9(x-x_1)(x-x_2)(x-x_3) = 9\left(x^3 - \underbrace{(x_1+x_2+x_3)}_{S}x^2 + (x_1x_2 + x_1x_3 + x_2x_3)x - \underbrace{x_1x_2x_3}_{P}\right)$$

$$-9S = -45 \Rightarrow \boxed{S = 5}$$

$$-9P = -40 \Rightarrow \boxed{P = \frac{40}{9}}$$

b)

$$\alpha - \beta,\ \alpha,\ \alpha + \beta$$

$$S = \alpha - \beta + \alpha + \alpha + \beta = 3\alpha = 5 \Rightarrow \boxed{\alpha = \frac{5}{3}}$$

$$\frac{5}{3} - \beta,\ \frac{5}{3},\ \frac{5}{3} + \beta$$

$$P = \left(\frac{5}{3} - \beta\right)\cdot\frac{5}{3}\cdot\left(\frac{5}{3} + \beta\right) = \frac{5}{3}\left(\frac{25}{9} - \beta^2\right) = \frac{40}{9} \Rightarrow \frac{125}{27} - \frac{5\beta^2}{3} = \frac{40}{9};$$

$$125 - 45\beta^2 = 120 \ ; \quad 5 = 45\beta^2 \Rightarrow \beta^2 = \frac{1}{9} \Rightarrow \boxed{\beta = \pm\frac{1}{3}}$$

Soluciones: $\frac{5}{3}$, $\frac{5}{3} \pm \frac{1}{3} \rightarrow \boxed{\frac{4}{3}, \frac{5}{3}, 2}$

Muestra 14 P2 #1

$$\left.\begin{array}{l} 2^3 + a\cdot 2^2 + b\cdot 2 - 4 = 0 \\ 1^3 + a\cdot 1^2 + b\cdot 1 - 4 = -6 \end{array}\right\} ; \quad \left.\begin{array}{l} 4a + 2b = -4 \\ a + b = -3 \end{array}\right\} ; \quad \left.\begin{array}{l} 2a + b = -2 \\ -a + b = -3 \end{array}\right\}$$

$$\boxed{a = 1} \rightarrow \boxed{b = -4}$$

M13 TZ1 P1 #1

$$\left.\begin{array}{l} 3\cdot 2^3 + a\cdot 3 + b = 2 \\ 3\cdot(-1)^2 + a\cdot(-1) + b = 5 \end{array}\right\} ; \quad \left.\begin{array}{l} 3a + b = -22 \\ -a + b = 2 \end{array}\right\}$$

$$2a = -24 \ ; \ \boxed{a = -12} \rightarrow \boxed{b = -10}$$

M14 TZ1 P1 #4

$$5x^3 + 48x^2 + 100x + 2 = a$$

$$5x^3 + 48x^2 + 100x + 2 - a = 0$$

$$r_1 + r_2 + r_3 = -\frac{48}{5}$$

$$r_1 r_2 r_3 = -\frac{2-a}{5}$$

$$\left(-\frac{48}{5}\right) + \left(-\frac{2-a}{5}\right) = 0 \ ; \quad -48 - 2 + a = 0 \ ; \ \boxed{a = 50}$$

M14 TZ2 P1 #4

a) $2x^2 + 4x - 1 = 0$

$$2(x-\alpha)(x-\beta) = 2\left(x^2 - (\alpha+\beta)x + \alpha\beta\right) = 2x^2 - 2(\alpha+\beta)x + 2\alpha\beta$$

$$\left.\begin{array}{l} -2(\alpha+\beta) = 4 \\ 2\alpha\beta = -1 \end{array}\right\} ; \quad \left.\begin{array}{l} \alpha + \beta = -2 \\ 2\alpha\beta = -1 \end{array}\right\}$$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-2)^2 - (-1) = \boxed{5}$$

b)

$$\left.\begin{array}{l} S = \alpha^2 + \beta^2 = 5 \\ P = \alpha^2\cdot\beta^2 = (\alpha\beta)^2 = (-1/2)^2 = \frac{1}{4} \end{array}\right\} \Rightarrow x^2 - 5x + \frac{1}{4} = 0$$

$$\boxed{4x^2 - 20x + 1 = 0}$$

M14 TZ1 P2#8a

$$(2x+B)^6 = \cdots + \binom{6}{1}(2x)^5 \cdot B^1 + \binom{6}{2}(2x)^4 \cdot B^2 + \cdots =$$

$$= \cdots + 192Bx^5 + 240B^2x^4 + \cdots$$

$$(3x+A)(2x+B)^6 = \cdots + (192BA + 720B^2)x^5 + \cdots$$

$$\boxed{192AB + 720B^2}$$

M14 TZ2 P2#5

$$(x-1)^3 = x^3 - 3x^2 + 3x - 1$$

$$\left(\frac{1}{x}+2x\right)^6 = \frac{1}{x^6} + 6\cdot\frac{1}{x^5}\cdot 2x + 15\cdot\frac{1}{x^4}\cdot(2x)^2 + 20\frac{1}{x^3}(2x)^3 + 15\frac{1}{x^2}(2x)^4 +$$

$$+ 6\frac{1}{x}\cdot(2x)^5 + (2x)^6 =$$

$$= x^{-6} + 12x^{-4} + 60x^{-2} + 160 + 240x^2 + 192x^4 + 64x^6$$

$$(x-1)^3\left(\frac{1}{x}+2x\right)^6 = \cdots + (-3x^2)\cdot 12x^{-4} + (-1)\cdot 60x^{-2} + \cdots =$$

$$= \cdots + (-36-60)x^{-2} + \cdots = \cdots \boxed{-96x^{-2}} \cdots$$

N14 P1#2

a)

$$2x^2 - 8x + 1 = 0$$

$$\alpha+\beta = -\frac{-8}{2} \longrightarrow \boxed{\alpha+\beta = 4}$$

$$\boxed{\alpha\cdot\beta = \frac{1}{2}}$$

b)

$$x^2 + px + q = 0$$

Raíces $\frac{2}{\alpha} + \frac{2}{\beta}$

$$S = \frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\beta+2\alpha}{\alpha\beta} = \frac{2(\alpha+\beta)}{\alpha\beta} = \frac{2\cdot 4}{1/2} = \boxed{16}$$

$$P = \frac{2}{\alpha}\cdot\frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{4}{1/2} = 8$$

$$\Rightarrow \boxed{x^2 - 16x + 8 = 0}$$

N14 P2#6

$$3x^3 + ax + 5a \qquad (a \in \mathbb{R})$$

$$3\cdot a^3 + a\cdot a + 5a = -7$$

$$3a^3 + a^2 + 5a + 7 = 0$$

$$a = -1 \;\begin{array}{|rrrr} 3 & 1 & 5 & 7 \\ & -3 & 2 & -7 \\ \hline 3 & -2 & 7 & \boxed{0} \end{array}$$

$$3a^2 - 2a + 7 = 0$$

$$a = \frac{2 \pm \sqrt{4-84}}{6} \notin \mathbb{R}$$

M15 TZ2 P1#4

$$(3-x)^4 = 3^4 - 4\cdot 3^3\cdot x + 6\cdot 3^2 x^2 - 4\cdot 3\cdot x^3 + x^4 = \boxed{x^4 - 12x^3 + 54x^2 - 108x + 81}$$

M15 TZ2 P2#3b

$$(x-5)^2 - 2|x-5| - 9 = 0$$

- Si $x > 5 \longrightarrow (x-5)^2 - 2(x-5) - 9 = 0$ ; $x^2 - 12x + 26 = 0$ ; $x = \begin{cases} \boxed{9'16} \\ \cancel{2'84} \text{ No es } >5 \end{cases}$
- Si $x < 5 \longrightarrow (x-5)^2 + 2(x-5) - 9 = 0$ ; $x^2 - 8x + 6 = 0$ ; $x = \begin{cases} \cancel{7'16} \text{ No es } <5 \\ \boxed{0'838} \end{cases}$