M00 P1

$$S_n = 3n^2 - 2n$$
$$- \; S_{n-1} = 3(n-1)^2 - 2(n-1)$$

$$a_n = 3n^2 - 2n - 3(n-1)^2 + 2(n-1) = \cancel{3n^2} - 2n - \cancel{3n^2} + 6n - 3 + \cancel{2n} - 2 =$$
$$= \boxed{6n-5}$$

También: $a_1 = S_1 = 3 \cdot 1^2 - 2 \cdot 1 = 1$

$$S_n = \frac{a_1 + a_n}{2} \cdot n \Rightarrow 3n^2 - 2n = \frac{1+a_n}{2} \cdot n \; ; \; 6n^2 - 4n = (1+a_n) \cdot n \; ;$$

$$6n - 4 = 1 + a_n \Rightarrow \boxed{a_n = 6n-5}$$

N00 P1

$85, 78, 71, \ldots$ $\quad a_n = 85 + (-7) \cdot (n-1) = 92 - 7n$

$a_n < 0 \Rightarrow 92 - 7n < 0 \; ; \; 92 < 7n \; ; \; \frac{92}{7} < n \; ; \; n > 13'\ldots$

Es decir, que son positivos los 13 primeros.

$$S_{13} = \frac{a_1 + a_{13}}{2} \cdot 13 = \frac{85 + 92 - 7 \cdot 13}{2} \cdot 13 = \boxed{559}$$

N00 P1

$$\left.\begin{array}{l} S_\infty = 13'5 \\ S_3 = 13 \end{array}\right| \quad \left.\begin{array}{l} \dfrac{a_1}{1-r} = 13'5 \\ \dfrac{a_3 \cdot r - a_1}{r-1} = 13 \end{array}\right\} \quad \left.\begin{array}{l} a_1 = 13'5(1-r) \\ \dfrac{a_1 \cdot r^{3-1} \cdot r - a_1}{r-1} = 13 \end{array}\right\} \quad \left.\begin{array}{l} a_1 = 13'5(1-r) \\ \dfrac{a_1(r^3-1)}{r-1} = 13 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \frac{13'5(1-r)(r^3-1)}{r-1} = 13 \; ; \quad \frac{13'5 \cdot \cancel{(1-r)}(r^3-1)}{-\cancel{(1-r)}} = 13 \; ; \quad 13'5(r^3-1) = -13 \; ;$$

$$r^3 - 1 = -\frac{13}{13'5} \; ; \quad r^3 = 1 - \frac{13}{13'5} = \frac{0'5}{13'5} = \frac{1}{27} \Rightarrow r = \sqrt[3]{1/27} = \frac{1}{3} \Rightarrow \boxed{a_1 = 9}$$

M01 P1

$$u_n = 3 \cdot 4^{n+1} \Rightarrow r = 4$$

$$S_n = \frac{u_n \cdot r - u_1}{r-1} = \frac{3 \cdot 4^{n+2} - 3 \cdot 4^2}{4-1} = \boxed{4^{n+2} - 16} = \boxed{16 \cdot (4^n - 1)}$$

N01 P1

$$1 + \left(\frac{2x}{3}\right) + \left(\frac{2x}{3}\right)^2 + \left(\frac{2x}{3}\right)^3 + \cdots$$

Son términos de una progresión geométrica de razón: $r = \frac{2x}{3}$

Converge si $|r| < 1 \Rightarrow \left|\frac{2x}{3}\right| < 1 \; ; \; -1 < \frac{2x}{3} < 1 \Rightarrow \boxed{-\frac{3}{2} < x < \frac{3}{2}}$

$$S_\infty = \frac{1}{1 - \frac{2x}{3}} = \frac{3}{3-2x}$$

$$x = 1'2 \Rightarrow S_\infty = \frac{3}{3 - 2 \cdot 1'2} = \boxed{5}$$

M02 P1

$2 + 5 + 8 + \cdots$

Los términos pertenecen a la progresión aritmética: $a_n = 2 + 3(n-1) = 3n-1$

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{2 + 3n - 1}{2} \cdot n = \boxed{\frac{(3n+1) \cdot n}{2}} = \boxed{\frac{3n^2+n}{2}}$$

$$S_n = 1365 \Rightarrow \frac{3n^2+n}{2} = 1365 \; ; \quad 3n^2 + n - 2730 = 0$$

$$n = \frac{-1 \pm \sqrt{1 + 32760}}{6} = \frac{-1 \pm 181}{6} \begin{cases} \boxed{30} \\ \cancel{-91/3} \end{cases}$$

N02 P1

$$\sum_{r=1}^{50} \ln(2^r) = \ln 2 + \ln 2^2 + \cdots + \ln 2^{50} =$$
$$= \ln\left[2 \cdot 2^2 \cdots 2^{50}\right] = \ln 2^{1+2+\cdots+50} =$$
$$= (1+2+\cdots+50)\cdot \ln 2 = \frac{1+50}{2}\cdot 50 \cdot \ln 2 = \boxed{1275 \ln 2}$$

N02 P2

a)
$u_0 = 1$
$u_1 = 2$
$$u_{n+1} = 3u_n - 2u_{n-1} \Rightarrow \begin{aligned} u_2 &= 3u_1 - 2u_0 = 6-2 = 4 \\ u_3 &= 3u_2 - 2u_1 = 12-4 = 8 \\ u_4 &= 3u_3 - 2u_2 = 24-8 = 16 \end{aligned}$$

b) La sucesión es: $1, 2, 4, 8, 16, \ldots \Rightarrow \boxed{u_n = 2^n}$

$u_{n+1} = 2^{n+1}$

$3u_n - 2u_{n-1} = 3\cdot 2^n - 2\cdot 2^{n-1} = 3\,2^n - 2^n = 2\cdot 2^n = 2^{n+1}$ ✓

M03 P1

a)
$$\left.\begin{aligned} a_1 + a_2 &= 15 \\ S_\infty &= 27 \end{aligned}\right\} \quad \left.\begin{aligned} a_1 + a_1 r &= 15 \\ \frac{a_1}{1-r} &= 27 \end{aligned}\right\} \quad \left.\begin{aligned} a_1(1+r) &= 15 \\ a_1 &= 27(1-r) \end{aligned}\right\} \longrightarrow 27(1-r)(1+r) = 15 \;;$$

$27(1-r^2) = 15 \;;\quad 1 - r^2 = \frac{15}{27} \;;\quad -r^2 = \frac{15}{27} - 1 \;;\quad -r^2 = -\frac{12}{27} \;;\quad r^2 = \frac{4}{9} \Rightarrow r = \boxed{\frac{2}{3}}$; $\cancel{-\frac{2}{3}}$

Dice que todos sus términos son positivos

b) $r = \frac{2}{3} \Rightarrow \boxed{a_1 = 9}$

N03 P1

$2,\ a-b,\ 2a+b+7,\ a-3b$

$a - b - 2 = 2a + b + 7 - (a-b) = a - 3b - (2a+b+7) \Rightarrow$

$$\left.\begin{aligned} a-b-2 &= 2a+b+7-a+b \\ a-b-2 &= a-3b-2a-b-7 \end{aligned}\right\} ;\quad \begin{aligned} -3b &= 9 \longrightarrow \boxed{b=-3} \\ 2a+3b &= -5 \longrightarrow 2a - 9 = -5 \Rightarrow \boxed{a=2} \end{aligned}$$

M04 P1

$$\begin{aligned} PA:&\ a, 1, b \rightarrow 1-a = b-1 \\ PG:&\ 1, a, b \rightarrow \frac{a}{1} = \frac{b}{a} \end{aligned} \quad \left.\begin{aligned} a+b &= 2 \\ a^2 &= b \end{aligned}\right\} \longrightarrow a + a^2 = 2 \;;$$

$a^2 + a - 2 = 0 \;;\quad a = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1\pm 3}{2}$; $\cancel{1 \Rightarrow b = 1}$ ← Dice que $a \neq b$; $-2 \Rightarrow b = 4$; $\boxed{\begin{aligned} a &= -2 \\ b &= 4 \end{aligned}}$

M04 P1

$$\left.\begin{aligned} a_1 + a_2 &= 6 \\ S_\infty &= 8 \end{aligned}\right\} \quad \left.\begin{aligned} a_1 + a_1 r &= 6 \\ \frac{a_1}{1-r} &= 8 \end{aligned}\right\} \quad \left.\begin{aligned} a_1(1+r) &= 6 \\ a_1 &= 8(1-r) \end{aligned}\right\} \longrightarrow 8(1-r)(1+r) = 6 \;;$$

$8(1-r^2) = 6 \;;\quad 8 - 8r^2 = 6 \;;\quad 2 = 8r^2 \;;\quad 1 = 4r^2 \;;\quad \frac{1}{4} = r^2 \;;$

$r = \pm\sqrt{\frac{1}{4}} =$ $\cancel{1/2}$ ← Dice que r es negativa ; $\boxed{-1/2} \Rightarrow \boxed{a_1 = 12}$

M04 P2

$$\ln x^2 + \ln \frac{x^2}{y} + \ln \frac{x^2}{y^2} + \cdots + \ln \frac{x^2}{y^{34}} = \ln\left[x^2 \cdot \frac{x^2}{y} \cdot \frac{x^2}{y^2} \cdots \frac{x^2}{y^{34}}\right] =$$
$$= \ln\left(\frac{(x^2)^{35}}{y^{1+2+\cdots+34}}\right) = \ln\left(\frac{x^{70}}{y^{\frac{1+34}{2}\cdot 34}}\right) = \boxed{\ln \frac{x^{70}}{y^{595}}}$$

N04 P1

$S_n = 2n^2 - n$

a) $a_1 = S_1 = 2\cdot 1^2 - 1 = \boxed{1}$

$a_2 = S_2 - S_1 = (2\cdot 2^2 - 2) - (2\cdot 1^2 - 1) = 6 - 1 = \boxed{5}$

$a_3 = S_3 - S_2 = (2\cdot 3^2 - 3) - (2\cdot 2^2 - 2) = 15 - 6 = \boxed{9}$

b) $a_n = S_n - S_{n-1} = (2n^2 - n) - \left(2\cdot(n-1)^2 - (n-1)\right) = 2n^2 - \cancel{n} - 2(n-1)^2 + \cancel{n} - 1 =$

$= \cancel{2n^2} - \cancel{2n^2} + 4n - 2 - 1 = \boxed{4n-3}$

M05 P1

$S_n = 4n^2 - 2n$

$u_n = S_n - S_{n-1} = (4n^2 - 2n) - \left(4(n-1)^2 - 2(n-1)\right) = 4n^2 - 2n - 4(n-1)^2 + 2(n-1) =$

$= \cancel{4n^2} - \cancel{2n} - \cancel{4n^2} + 8n - 4 + \cancel{2n} - 2 = \boxed{8n-6}$

$u_2 = 8\cdot 2 - 6 = 10$
$u_m = 8m - 6$
$u_{32} = 8\cdot 32 - 6 = 250$

$\xrightarrow{P.G.} \frac{8m-6}{10} = \frac{250}{8m-6} \Rightarrow (8m-6)^2 = 2500$;

$64m^2 - 96m + 36 = 2500$; $64m^2 - 96m - 2464 = 0$

$2m^2 - 3m - 77 = 0$

$m = \frac{3 \pm \sqrt{9 + 616}}{4} = \frac{3 \pm 25}{4} = \begin{cases} \boxed{7} \\ \cancel{-5.5} \end{cases}$ ← Tiene que ser un nº entero

M05 P2

$\sum_{r=1}^{n} (r+1)2^{r-1} = n\cdot 2^n \longrightarrow 2 + 3\cdot 2 + 4\cdot 2^2 + 5\cdot 2^3 + \cdots + (n+1)2^{n-1} = n\cdot 2^n$

N09 P1

$n=1$ $\quad 2 = 1\cdot 2^1$ ✓

$n=K$ Suponiendo cierto que : $2 + 3\cdot 2 + \cdots + (K+1)\cdot 2^{K-1} = K\cdot 2^K$, vamos a demostrar que : $2 + 3\cdot 2 + \cdots + (K+1)\cdot 2^{K-1} + (K+2)\cdot 2^K = (K+1)\cdot 2^{K+1}$:

$\underbrace{2 + 3\cdot 2 + \cdots + (K+1)2^{K-1}}_{K\cdot 2^K} + (K+2)\cdot 2^K = K\cdot 2^K + (K+2)\cdot 2^K = (K+K+2)2^K =$

$= (2K+2)\cdot 2^K = 2(K+1)\cdot 2^K = (K+1)\cdot 2^{K+1}$ ✓

M05 P2

P.G. : $a, ar, ar^2 \longrightarrow S_\infty = 18 = \frac{a}{1-r}$

P.A. : $a, \ldots\ldots, ar, \ldots, ar^2$

ar: $a + d\cdot 10$; ar^2: $a + d\cdot 15$

$\left.\begin{array}{l} 18 = \frac{a}{1-r} \\ ar = a + 10d \\ ar^2 = a + 15d \end{array}\right\}$ $\left.\begin{array}{l} a = 18(1-r) \\ a(r-1) = 10d \\ a(r^2-1) = 15d \end{array}\right\}$ $\left.\begin{array}{l} 18(1-r)(r-1) = 10d \\ 18(1-r)(r^2-1) = 15d \end{array}\right\}$ $\left.\begin{array}{l} 9(1-r)(r-1) = 5d \\ 6(1-r)(r^2-1) = 5d \end{array}\right\} \Rightarrow$

$\Rightarrow 9(1-r)(r-1) = 6(1-r)(r^2-1)$; $3(1-r)(r-1) = 2(1-r)(r+1)(r-1)$;

$3(1-r)(r-1) - 2(1-r)(r+1)(r-1) = 0$; $(1-r)(r-1)\left(3 - 2(r+1)\right) = 0$;

$(1-r)(r-1)(1-2r) = 0$
- $1-r = 0 \Rightarrow \cancel{r=1}$ Dicen que todos los términos son diferentes
- $r-1 = 0 \Rightarrow \cancel{r=1}$
- $1-2r = 0 \Rightarrow \boxed{r = \frac{1}{2}} \Rightarrow 9(1-\frac{1}{2})(\frac{1}{2}-1) = 5d \Rightarrow$

$\Rightarrow d = \frac{9\cdot\frac{1}{2}\cdot\frac{-1}{2}}{5} = \boxed{-\frac{9}{20}}$

N05 P1

$$\sum_{n=1}^{15} a_n^2 = \sum_{n=1}^{15} (\ln x^n)^2 = \sum_{n=1}^{15} (n \ln x)^2 = \sum_{n=1}^{15} n^2 \cdot \ln^2 x = \ln^2 x \cdot \sum_{n=1}^{15} n^2 =$$

$$= \ln^2 x \cdot (1+2^2+3^2+ \dots +15^2) = \boxed{1240 \ln^2 x}$$

N05 P2

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1} \longrightarrow \frac{1}{3}+\frac{1}{15}+\dots+\frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

a)

$n=1$ $\qquad \frac{1}{3} = \frac{1}{2\cdot 1+1}$ ✓

$n=K$ Suponiendo cierto que $\frac{1}{3}+\frac{1}{15}+\dots+\frac{1}{(2K-1)(2K+1)} = \frac{K}{2K+1}$ vamos a demostrar que $\frac{1}{3}+\frac{1}{15}+\dots+\frac{1}{(2K-1)(2K+1)}+\frac{1}{(2(K+1)-1)(2(K+1)+1)} = \frac{K+1}{2(K+1)+1}$:

$$\underbrace{\frac{1}{3}+\frac{1}{15}+\dots+\frac{1}{(2K-1)(2K+1)}}_{\frac{K}{2K+1}}+\frac{1}{(2(K+1)-1)(2(K+1)+1)} = \frac{K}{2K+1}+\frac{1}{(2K+1)(2K+3)} =$$

$$= \frac{K(2K+3)+1}{(2K+1)(2K+3)} = \frac{2K^2+3K+1}{(2K+1)(2K+3)} = \frac{(K+1)\cancel{(2K+1)}}{\cancel{(2K+1)}(2K+3)} = \frac{K+1}{2K+3} = \frac{K+1}{2(K+1)+1} \checkmark$$

$$\begin{array}{r|rrr} & 2 & 3 & 1 \\ -1 & & -2 & -1 \\ \hline & 2 & 1 & \boxed{0} \end{array}$$

b) $K=n+1$ $\qquad \frac{1}{3}+\frac{1}{15}+\dots+\frac{1}{(2n+1)(2n+3)} = \frac{n+1}{2(n+1)+1} = \boxed{\frac{n+1}{2n+3}}$ ✓

M06 P1

$$\left.\begin{array}{l} a_2 = 7 \\ S_5 = 50 \end{array}\right| \quad \left.\begin{array}{l} a_1+d = 7 \\ \frac{a_1+a_5}{2}\cdot 5 = 50 \end{array}\right\} \quad \left.\begin{array}{l} a_1+d = 7 \\ \frac{a_1+a_1+4d}{2}\cdot 5 = 50 \end{array}\right\} \quad \left.\begin{array}{l} a_1 = 7-d \\ (2a_1+4d)5 = 100 \end{array}\right\} \Rightarrow$$

$$\Rightarrow (14-2d+4d)5 = 100 \quad , \quad 14+2d = 20 \quad ; \quad 2d = 6 \quad ; \quad \boxed{d=3}$$

N06 P1

$$\left.\begin{array}{l} S_\infty = 32 \\ S_4 = 30 \end{array}\right\} \quad \left.\begin{array}{l} \frac{a_1}{1-r} = 32 \\ \frac{a_4\cdot r-a_1}{r-1} = 30 \end{array}\right\} \quad \left.\begin{array}{l} a_1 = 32(1-r) \\ a_1 r^3\cdot r-a_1 = 30(r-1) \end{array}\right\} \quad \left.\begin{array}{l} a_1 = 32(1-r) \\ a_1(r^4-1) = 30(r-1) \end{array}\right\} \Rightarrow$$

$$\Rightarrow 32(1-r)(r^4-1) = 30(r-1) \quad ; \quad -16(r-1)(r^4-1) = 15(r-1) \quad ; \quad 0 = 15(r-1)+16(r-1)(r^4-1)$$

$0 = (r-1)(15+16(r^4-1))$ $\quad ; \quad 0 = (r-1)\cdot(-1+16r^4)$ $<$ $\cancel{r=1}$ La suma S_∞ sería ∞ ; $-1+16r^4=0 \Rightarrow$

$\Rightarrow 16r^4 = 1 \quad , \quad r = \pm\sqrt[4]{\frac{1}{16}} =$ $<$ $\frac{1}{2} \rightarrow a_1 = 16$; $\cancel{-\frac{1}{2}}$ Dicen que todos los términos son positivos

$$S_\infty - S_8 = \frac{16}{1-\frac{1}{2}} - \frac{16\left(\frac{1}{2}\right)^7\cdot\frac{1}{2}-16}{\frac{1}{2}-1} = 32 - \frac{\frac{1}{16}-16}{-1/2} = \cancel{32}+\frac{1}{8}-\cancel{32} = \boxed{\frac{1}{8}}$$

N06 P2

a) $1\cdot 1! + 2\cdot 2! + \dots + n\cdot n! = (n+1)! - 1$

$n=1$ $\quad 1\cdot 1! = (1+1)! - 1 \quad , \quad 1 = 2-1$ ✓

$n=K$ Suponiendo cierto que $1\cdot 1! + 2\cdot 2! + \dots + K\cdot K! = (K+1)! - 1$, vamos a demostrar que $1\cdot 1! + 2\cdot 2! + \dots + K\cdot K! + (K+1)(K+1)! = (K+2)! - 1$:

$\underbrace{1\cdot 1! + 2\cdot 2! + \dots + K\cdot K!} + (K+1)(K+1)! = (K+1)! - 1 + (K+1)\cdot(K+1)! =$

$= (K+1)! \cdot [1 + K+1] - 1 = (K+1)!\cdot(K+2) - 1 = (K+2)! - 1$ ✓

b) $(n+1)! - 1 > 10^9 \quad ; \quad (n+1)! > 10^9 + 1 = 1.000.000.001$

$12! = 479.001.600$

$13! = 6.227.020.800 > 10^9+1 \Rightarrow \boxed{n=12}$

M07 P1

$-6+1+8+15+\dots$

Los sumandos pertenecen a la progresión aritmética $a_n = -6+7(n-1) = 7n-13$

$S_n > 10000$

$\frac{-6+7n-13}{2}\cdot n > 10000 \quad ; \quad (7n-19)n > 20000 \quad , \quad 7n^2 - 19n - 20000 > 0$

$7n^2 - 19n - 20000 = 0$

$n = \frac{19 \pm \sqrt{361 + 560000}}{14} = \frac{19 \pm \sqrt{560361}}{14}$ — $54'83$; $\cancel{-52'1}$

Signo $7n^2-19n-20000$	1	54'83
	$-$	$+$

$\boxed{n=55}$

M07 P1

$\sum_{K=1}^{\infty} 2\cdot(4-3x)^K$

a) Debe ser $-1 < 4-3x < 1 \quad ; \quad -5 < -3x < -3 \quad ; \quad \frac{5}{3} > x > 1 \Rightarrow \boxed{x \in (1, \frac{5}{3})}$

b) $x = 1'2 \longrightarrow a_n = 2\cdot(4-3\cdot 1'2)^n \quad ; \quad a_n = 2\cdot 0'4^n \longrightarrow a_1 = 0'8 \quad r = 0'4$

$S_n > 1'328 \Rightarrow \frac{a_n\cdot r - a_1}{r-1} > 1'328 \quad ; \quad \frac{2\cdot 0'4^n\cdot 0'4 - 0'8}{0'4-1} > 1'328 \quad ;$

$\frac{0'8\cdot 0'4^n - 0'8}{-0'6} > 1'328 \quad ; \quad 0'8\cdot 0'4^n - 0'8 < -0'7968 \quad , \quad 0'8\cdot 0'4^n < 0'0032 \quad ,$

$0'4^n < \frac{0'0032}{0'8} \quad ; \quad 0'4^n < 0'004 \quad ; \quad \ln 0'4^n < \ln 0'004 \quad ; \quad n\ln 0'4 < \ln 0'004 \quad ;$

$n > \frac{\ln 0'004}{\ln 0'4} = 6'03 \Rightarrow \boxed{n=7}$

Se voltea la desigualdad porque $\ln 0'4$ es negativo

También :

$0'4^1 = 0'4 \not< 0'004$

$0'4^2 = 0'16 \not< 0'004$

$0'4^6 = 0'004096 \not< 0'004$

$0'4^7 = 0'0016384 < 0'004 \Rightarrow \boxed{n=7}$.

M07 P2

$$12^n + 2\cdot 5^{n-1} = \dot{7}$$

$\uparrow$ múltiplo de 7

n=1 $\quad 12^1 + 2\cdot 5^{1-1} = 12+2 = 14 = \dot{7}$ ✓

n=K Suponiendo que $12^K + 2\cdot 5^{K-1}$ es un múltiplo de 7 vamos a demostrar que $12^{K+1} + 2\cdot 5^K$ también es un múltiplo de 7:

$$12^{K+1} + 2\cdot 5^K = 12\cdot 12^K + 2\cdot 5^K = 12\cdot\left(12^K + 2\cdot 5^{K-1} - 2\cdot 5^{K-1}\right) + 2\cdot 5^K =$$
$$= 12\cdot(12^K + 2\cdot 5^{K-1}) - 24\cdot 5^{K-1} + 2\cdot 5^K =$$
$$= 12\cdot(12^K + 2\cdot 5^{K-1}) - 2\cdot 5^{K-1}\cdot(12-5) =$$
$$= 12\cdot\underbrace{(12^K + 2\cdot 5^{K-1})}_{\dot{7}} - \underbrace{2\cdot 5^{K-1}\cdot 7}_{\dot{7}} = \dot{7} \ ✓$$

N07 P1

$$\left.\begin{array}{l} a_1 = 18 \\ a_4 = -\frac{2}{3}\end{array}\right| \longrightarrow \quad 18\cdot r^3 = -\frac{2}{3} \ ; \ r^3 = \frac{-2}{54} \ ; \ r^3 = -\frac{1}{27} \Rightarrow r = -\frac{1}{3}$$

$$a_n = 18\cdot\left(-\frac{1}{3}\right)^{n-1}$$

a) $$S_n = \frac{a_n\cdot r - a_1}{r-1} = \frac{18\cdot\left(-\frac{1}{3}\right)^{n-1}\cdot\left(-\frac{1}{3}\right) - 18}{-\frac{1}{3}-1} = \frac{18\left(\left(-\frac{1}{3}\right)^n - 1\right)}{-4/3} =$$
$$= -\frac{27}{2}\left(\left(-\frac{1}{3}\right)^n - 1\right) = \boxed{\frac{27}{2} - \frac{27}{2}\left(-\frac{1}{3}\right)^n}$$

b) $$S_\infty = \frac{18}{1-\left(-\frac{1}{3}\right)} = \frac{18}{4/3} = \boxed{\frac{27}{2}}$$

Muestra 08 P1

(i) P.A: $1, 1+p, 1+2p, 1+3p, \dots$ $\qquad$ P.G: $1, p, p^2, p^3, \dots$

(ii) $1+2p+1+3p = p^2+p^3 \ ; \quad 0 = p^3+p^2-5p-2$

$$\begin{array}{r|rrrr} & 1 & 1 & -5 & -2 \\ 2 & & 2 & 6 & 2 \\ \hline & 1 & 3 & 1 & \boxed{0} \end{array}$$

$p^2+3p+1=0 \qquad p = \dfrac{-3\pm\sqrt{9-4}}{2} = \dfrac{-3\pm\sqrt{5}}{2}$ $\qquad \boxed{p = \begin{cases} 2 \\ \frac{-3\pm\sqrt{5}}{2}\end{cases}}$

(iii) Para que exista S_∞, la razón debe ser $-1<p<1$
El único valor de p es $\boxed{p = \frac{-3+\sqrt{5}}{2}}$

(iv) $p = \dfrac{-3+\sqrt{5}}{2}$

$$S_{20} = \frac{a_1+a_{20}}{2}\cdot 20 = \frac{1+1+19\cdot\left(\frac{-3+\sqrt{5}}{2}\right)}{2}\cdot 20 = \left(2+\frac{19(-3+\sqrt{5})}{2}\right)\cdot 10 =$$
$$= 20 + 95(-3+\sqrt{5}) = \boxed{-265+95\sqrt{5}}$$

Muestra 08 P2

$\boxed{a_n = 5000\cdot 1'063^n}$ $\qquad n=5 \rightarrow a_5 = 5000\cdot 1'063^5 = \boxed{6786'35\ \$}$

$5000\cdot 1'063^n > 10.000 \ ; \ 1'063^n > 2 \ ; \ n\ln 1'063 > \ln 2 \Rightarrow n > \dfrac{\ln 2}{\ln 1'063} = 11'35$

$\boxed{n = 12 \text{ años}}$

Muestra 08
P1

$5^n + 9^n + 2 = \dot{4}$

$n=1$ $\quad 5^1 + 9^1 + 2 = 16 = \dot{4}$ ✓

$n=K$ Suponiendo que $5^K + 9^K + 2$ es un múltiplo de 4 vamos a demostrar que $5^{K+1} + 9^{K+1} + 2$ también lo es:

$$5^{K+1} + 9^{K+1} + 2 = 5 \cdot 5^K + 9^{K+1} + 2 = 5(5^K + 9^K + 2 - 9^K - 2) + 9^{K+1} + 2 =$$
$$= 5(5^K + 9^K + 2) - 5 \cdot 9^K - 10 + 9^{K+1} + 2 =$$
$$= 5(5^K + 9^K + 2) + 9^K(-5+9) - 8 =$$
$$= 5\underbrace{(5^K + 9^K + 2)}_{\dot{4}} + 9^K \cdot 4 - 8 = \dot{4} \checkmark$$

M08
P1

$r = 2^x$

a) $\quad |2^x| < 1 \Rightarrow 2^x < 1 \Rightarrow \boxed{x < 0}$

b) $\quad a_1 = 35$, $S_\infty = 40$ $\quad \dfrac{35}{1-2^x} = 40 \;;\; 35 = 40 - 40 \cdot 2^x \;;\; 40 \cdot 2^x = 5 \;;\; 2^x = \dfrac{1}{8} \;;\; \boxed{x=-3}$

M08
P1

$27, -9, 3, -1, \ldots \rightarrow r = -\dfrac{1}{3}$

$$S_\infty = \frac{27}{1-\left(-\frac{1}{3}\right)} = \frac{27}{4/3} = \boxed{\frac{81}{4}}$$

M08
P1

$$a + ar + ar^2 + \ldots + a \cdot r^{n-1} = \frac{a(1-r^n)}{1-r}$$

$n=1$ $\quad a = \dfrac{a(\cancel{1-r^1})}{\cancel{1-r}}$ ✓

$n=K$ Suponiendo cierto de $a + ar + \ldots + ar^{K-1} = \dfrac{a(1-r^K)}{1-r}$ vamos a demostrar que $a + ar + \ldots + ar^{K-1} + a \cdot r^K = \dfrac{a(1-r^{K+1})}{1-r}$:

$$(a + ar + \ldots + ar^{K-1}) + a \cdot r^K = \frac{a(1-r^K)}{1-r} + a \cdot r^K = \frac{a(1-r^K) + ar^K(1-r)}{1-r} =$$
$$= \frac{a - \cancel{ar^K} + \cancel{ar^K} - ar^{K+1}}{1-r} = \frac{a(1-r^{K+1})}{1-r} \checkmark$$

N08
P2

$a_1 = 2$, $r = 1'05$ $\quad a_n = 2 \cdot 1'05^{n-1}$

$a_n > 500 \;;\; 2 \cdot 1'05^{n-1} > 500 \;;\; 1'05^{n-1} > 250 \;,\; \ln 1'05^{n-1} > \ln 250 \;;$

$(n-1)\ln 1'05 > \ln 250 \;;\; n-1 > \dfrac{\ln 250}{\ln 1'05} \Rightarrow n > 1 + \dfrac{\ln 250}{\ln 1'05} = 114'17 \Rightarrow n = 115$

$\Downarrow$

$\boxed{a_{115} = 521}$

N08
P1

P.A. $a_1 = 1'5$, $a_n = 7'5$, $S_n = 81$

$$81 = \frac{1'5 + 7'5}{2} \cdot n \Rightarrow \boxed{n = 18}$$

$$a_{18} = 7'5 \longrightarrow 7'5 = 1'5 + 17d \Rightarrow \boxed{d = \frac{6}{17}}$$

M09
P1

$a, 2a, 3a, \dots na$

(i) media $= \dfrac{a + 2a + 3a + \dots + na}{n} = \dfrac{a(1+2+\dots+n)}{n} = \dfrac{a\,\frac{1+n}{2}\cdot \cancel{n}}{\cancel{n}} = \boxed{\dfrac{a(n+1)}{2}}$ ✓

(ii) $a = 4 \rightarrow 4, 8, 12, \dots, 4n \longrightarrow$ media $= \dfrac{4(n+1)}{2} = 2(n+1)$

$S_n >$ media $+100$

$$\frac{4+4n}{2} \cdot n > 2(n+1) + 100 \;;\; (2+2n)n > 2n + 2 + 100 \;;\; \cancel{2n} + 2n^2 > \cancel{2n} + 102$$

$$2n^2 > 102 \;;\; n^2 > 51 \;;\; n > \sqrt{51} = 7'14 \Rightarrow \boxed{n = 8}$$

N09
P1

a) $\left.\begin{array}{l} S_6 = 81 \\ S_{11} = 231 \end{array}\right\}$; $\left.\begin{array}{l} \frac{a_1 + a_6}{2} \cdot 6 = 81 \\ \frac{a_1 + a_{11}}{2} \cdot 11 = 231 \end{array}\right\}$; $\left.\begin{array}{l} a_1 + a_1 + 5d = 27 \\ a_1 + a_1 + 10d = 42 \end{array}\right\}$; $\left.\begin{array}{l} 2a_1 + 5d = 27 \\ 2a_1 + 10d = 42 \end{array}\right\}$

$5d = 15$

$\boxed{d = 3} \Rightarrow \boxed{a_1 = 6}$

b) $\left.\begin{array}{l} S_2 = 1 \\ S_4 = 5 \end{array}\right\}$; $\left.\begin{array}{l} \frac{a_2 \cdot r - a_1}{r-1} = 1 \\ \frac{a_4 \cdot r - a_1}{r-1} = 5 \end{array}\right\}$; $\left.\begin{array}{l} a_1 r^2 - a_1 = r - 1 \\ a_1 r^4 - a_1 = 5(r-1) \end{array}\right\}$;

$\left.\begin{array}{l} a_1(r^2 - 1) = r - 1 \\ a_1(r^4 - 1) = 5(r-1) \end{array}\right\} \longleftrightarrow a_1 = \dfrac{r-1}{r^2-1} \longrightarrow \dfrac{r-1}{r^2-1}(r^4 - 1) = 5(r-1)$;

$$\frac{\cancel{(r-1)}(r^2+1)\cancel{(r^2-1)}}{\cancel{r^2-1}} = 5\cancel{(r-1)} \Rightarrow r^2 + 1 = 5 \;;\; r^2 = 4 \;;\; r = \boxed{2} \Rightarrow \boxed{a_1 = \frac{1}{3}}$$

$\cancel{-2}$ Dice que todos los términos son positivos

c) P.A: $a_n = 6 + 3(n-1) = 3n + 3 = 3(n+1)$

P.G: $b_n = \frac{1}{3} \cdot 2^{n-1}$

$$a_n \cdot b_n = \cancel{3}(n+1) \cdot \frac{1}{\cancel{3}} \cdot 2^{n-1} = \boxed{(n+1)\, 2^{n-1}}$$ ✓

M10
P1

(i) $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n \times (n+1) = \frac{1}{3}\left(n \times (n+1) \times (n+2)\right)$

(ii) $n = 4$: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 = 40$

$\frac{1}{3} \cdot 4 \cdot 5 \cdot 6 = 40$ ✓

M10
P1

$u_n = 2^n + 3$

$u_1 = 5$ ✓ $u_2 = 7$ ✓ $u_3 = 11$ ✓ $u_4 = 19$ ✓ $u_5 = 35$ * no es primo.

M10
P1 TZ2
#11

$5 \cdot 7^n + 1 = \dot{6} \quad (n \in \mathbb{Z}^+)$

$\underline{n=1}$ $\quad 5 \cdot 7^1 + 1 = 36 = \dot{6}$ ✓

$\underline{n=K}$ Suponiendo cierto que $5 \cdot 7^K + 1$ es divisible entre 6, demostraremos que $5 \cdot 7^{K+1} + 1$ también lo es:

$$5 \cdot 7^{K+1} + 1 = 5 \cdot 7^K \cdot 7 + 1 = 7 \cdot 5 \cdot 7^K + 1 = 7 \cdot (5 \cdot 7^K + 1 - 1) + 1 =$$

$$= 7 \cdot (5 \cdot 7^K + 1) - 7 + 1 = \underbrace{\underbrace{7 \cdot \underbrace{(5 \cdot 7^K + 1)}_{\dot{6}}}_{\dot{6}} - 6}_{\dot{6}}$$ ✓

M10
P2 TZ1
#6

$100 + 101 + \cancel{102} + 103 + 104 + \cancel{105} + \ldots + 998 + \cancel{999}$

$$\sum_{n=100}^{999} n = \frac{100 + 999}{2} \cdot 900 = 494550$$

$$102 + 105 + \ldots + 999 = \sum_{n=1}^{300} (3n + 99) = \frac{999 + 102}{2} \cdot 300 = 165150$$

$$S = 494550 - 165150 = \boxed{329400}$$

M10
P2 TZ2
#1

$8, 26, 44, \ldots$

a) $a_n = 8 + 18(n-1) = \boxed{18n - 10}$

b) $S_n = \boxed{\sum_{K=1}^{n} (18K - 10)}$

c) $S_{15} = \sum_{K=1}^{15} (18K - 10) = \frac{(18 \cdot 15 - 10) + 8}{2} \cdot 15 = \boxed{2010}$

N10
P1
#6

$S_n = \frac{7^n - a^n}{7^n} \quad (a > 0)$

a) $$u_n = S_n - S_{n-1} = \frac{7^n - a^n}{7^n} - \frac{7^{n-1} - a^{n-1}}{7^{n-1}} = \frac{7^n - a^n - 7(7^{n-1} - a^{n-1})}{7^n} =$$

$$= \frac{\cancel{7^n} - a^n - \cancel{7^n} + 7a^{n-1}}{7^n} = \boxed{\frac{a^{n-1}(7-a)}{7^n}}$$

b) $\underline{\text{También}}$:

$$u_1 = \frac{7^1 - a^1}{7^1} = \boxed{\frac{7-a}{7}}$$

$$u_2 = \frac{7^2 - a^2}{7^2} - \frac{7-a}{7} = \frac{\cancel{49} - a^2 - \cancel{49} + 7a}{49} = \frac{(7-a)a}{49}$$

$$\Rightarrow r = \frac{u_2}{u_1} = \frac{\frac{(\cancel{7-a})a}{49}}{\frac{\cancel{7-a}}{7}} = \boxed{\frac{a}{7}}$$

$$u_n = u_1 \cdot r^{n-1} = \frac{7-a}{7} \cdot \left(\frac{a}{7}\right)^{n-1} = \frac{(7-a) \cdot a^{n-1}}{7^n}$$ ✓

c) $-1 < \frac{a}{7} < 7 \Rightarrow$ Para $a \in (-7, 7)$ existirá la suma infinita

Como $a > 0 \Rightarrow \boxed{a \in (0, 7)}$

$$S_\infty = \frac{u_1}{1-r} = \frac{\frac{7-a}{7}}{1 - \frac{a}{7}} = \frac{\cancel{\frac{7-a}{7}}}{\cancel{\frac{7-a}{7}}} = \boxed{1}$$

N10 P1 #5

P.A. $\left.\begin{array}{l}\frac{S_{10}}{10}=6\\ \frac{S_{20}}{20}=16\end{array}\right\} \rightarrow \left.\begin{array}{l}\frac{\frac{a_1+a_{10}}{2}\cdot\cancel{10}}{\cancel{10}}=6\\ \frac{\frac{a_1+a_{20}}{2}\cdot\cancel{20}}{\cancel{20}}=16\end{array}\right\} \rightarrow \left.\begin{array}{l}a_1+a_{10}=12\\ a_1+a_{20}=32\end{array}\right\} \rightarrow$

$\rightarrow \left.\begin{array}{l}a_1+a_1+9d=12\\ a_1+a_1+19d=32\end{array}\right\} \rightarrow \left.\begin{array}{l}2a_1+9d=12\\ 2a_1+19d=32\end{array}\right\}$

$10d=20$

$\boxed{d=2} \rightarrow \boxed{a_1=-3}$

$a_{15}=a_1+14d=-3+14\cdot 2=\boxed{25}$

M11 P1 TZ1 #3

a) P.G. $\left.\begin{array}{l}u_1=27\\ S_\infty=\frac{81}{2}\end{array}\right\} \rightarrow \frac{81}{2}=\frac{27}{1-r}$; $81-81r=54$; $27=81r$; $\boxed{r=\frac{1}{3}}$

$u_n=27\cdot\left(\frac{1}{3}\right)^{n-1}$

b) P.A. $v_2=u_2=27\cdot\frac{1}{3}=9$; $v_4=u_4=27\cdot\left(\frac{1}{3}\right)^3=1$ $\left\{\begin{array}{l}v_2=v_1+d\\ v_4=v_1+3d\end{array}\right\} \rightarrow \left.\begin{array}{l}9=v_1+d\\ 1=v_1+3d\end{array}\right\}$

$-8=2d$

$\boxed{d=-4} \rightarrow \boxed{v_1=13}$

$v_n=13+(-4)(n-1)=17-4n$

$\sum_{n=1}^{N} v_n>0 \Rightarrow \frac{v_1+v_N}{2}\cdot N>0$; $\frac{13+17-4N}{2}\cdot N>0$; $\frac{(30-4N)\cdot N}{2}>0 \Rightarrow$

$\Rightarrow 30-4N>0$; $N>\frac{30}{4}=7'5 \Rightarrow \boxed{N=7}$

M11 P1 TZ2 #10

P.A. $a_n=a+d\cdot(n-1) \longrightarrow \begin{array}{l}a_3=a+2d\\ a_4=a+3d\\ a_7=a+6d\end{array}$ $(d\neq 0)$

P.G $a+2d,\ a+3d,\ a+6d$

$r=\frac{a+6d}{a+3d}=\frac{a+3d}{a+2d} \Rightarrow \cancel{a^2}+2ad+\cancel{6ad}+12d^2=\cancel{a^2}+\cancel{6ad}+9d^2$

$2a\cancel{d}=-3d^{\cancel{2}} \Rightarrow a=\frac{-3d}{2}$ ✓

$r=\frac{\frac{-3d}{2}+3d}{\frac{-3d}{2}+2d}=\frac{-3d+6d}{-3d+4d}=\frac{3d}{d}=3$

$b_1=a+2d=\frac{-3d}{2}+2d=\frac{-3d+4d}{2}=\frac{d}{2}$

$b_4=\frac{d}{2}\cdot 3^3=\boxed{\frac{27d}{2}}$

$a_{16}=a+15d=-\frac{3d}{2}+15d=\frac{-3d+30d}{2}=\boxed{\frac{27d}{2}}$ ✓

M11
P2 TZ2
#2

$$\left.\begin{array}{l} u_4 = 7 \\ u_9 = 22 \end{array}\right| \Rightarrow \begin{array}{l} 22 = u_1 + 8d \\ 7 = u_1 + 3d \\ \hline 15 = \quad 5d \end{array} ; \quad d = 3 \rightarrow u_1 = -2 \qquad u_n = -2 + 3(n-1)$$

$$\boxed{u_n = 3n - 5}$$

$$S_n > 10000 \Rightarrow \frac{u_1 + u_n}{2} \cdot n > 10000 \quad ; \quad \frac{-2 + 3n - 5}{2} \cdot n > 10000 \quad ;$$

$$(3n-7)n > 20000 \quad ; \quad 3n^2 - 7n - 20000 > 0$$

$$3n^2 - 7n - 20000 = 0$$

$$n = \frac{7 \pm \sqrt{49 + 240000}}{6} \nearrow 82'8\ldots \Rightarrow \boxed{n = 83}$$

$$\searrow -80'4\ldots$$

M11
P2 TZ2
#13

$$1 + 2\left(\tfrac{1}{2}\right) + 3\left(\tfrac{1}{2}\right)^2 + \cdots + n\left(\tfrac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}$$

$\underline{\underline{n=1}}$ $\quad 1 = 4 - \frac{3}{1} = 1$ ✓

$\underline{\underline{n=K}}$ Suponiendo cierto que $1 + 2\left(\tfrac{1}{2}\right) + \cdots + K\left(\tfrac{1}{2}\right)^{K-1} = 4 - \frac{K+2}{2^{K-1}}$

demostraremos que $1 + 2\left(\tfrac{1}{2}\right) + \cdots + K\left(\tfrac{1}{2}\right)^{K-1} + (K+1)\left(\tfrac{1}{2}\right)^K = 4 - \frac{K+3}{2^K}$

$$1 + 2\left(\tfrac{1}{2}\right) + \cdots + (K+1)\left(\tfrac{1}{2}\right)^K = 4 - \frac{K+2}{2^{K-1}} + (K+1)\cdot\frac{1}{2^K} = 4 - \frac{1}{2^K}\big(2(K+2) - (K+1)\big) =$$

$$= 4 - \frac{1}{2^K}(2K + 4 - K - 1) = 4 - \frac{K+3}{2^K} \quad ✓$$

N11
P2 #7

a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2x}{x+1}\right)^n$

Debe ser $-1 < \frac{2x}{x+1} < 1$:

$$-1 < \frac{2x}{x+1} \quad ; \quad 0 < \frac{2x}{x+1} + 1 \quad ; \quad 0 < \frac{3x+1}{x+1} \qquad x \in (-\infty, -1) \cup \left(-\tfrac{1}{3}, +\infty\right)$$

$$\frac{2x}{x+1} < 1 \quad ; \quad \frac{2x}{x+1} - 1 < 0 \quad ; \quad \frac{x-1}{x+1} < 0 \qquad x \in (-1, 1)$$

$$\Rightarrow \boxed{x \in \left(-\tfrac{1}{3}, 1\right)}$$

b) $$S_\infty = \frac{u_1}{1-r} = \frac{\left(\frac{2x}{x+1}\right)^1}{1 - \frac{2x}{x+1}} = \frac{\frac{2x}{\cancel{x+1}}}{\frac{x+1-2x}{\cancel{x+1}}} = \boxed{\frac{2x}{1-x}}$$

N11
P2
#12

$$\text{P.A.} \quad \left.\begin{array}{l} a_1 = 8 \\ d = \frac{1}{4} \end{array}\right| \rightarrow a_n = 8 + \tfrac{1}{4}(n-1) = \frac{n - 1 + 32}{4} = \boxed{\frac{n+31}{4}}$$

$$S_{2n} = \frac{a_1 + a_{2n}}{2}\cdot 2n = \frac{8 + \frac{2n+31}{4}}{\cancel{2}}\cdot\cancel{2}n = \frac{32 + 2n + 31}{4}\cdot n = \frac{2n^2 + 63n}{4}$$

$$S_{3n} = \frac{a_1 + a_{3n}}{2}\cdot 3n = \frac{8 + \frac{3n+31}{4}}{2}\cdot 3n = \frac{32 + 3n + 31}{8}\,3n = \frac{9n^2 + 189n}{8}$$

$$S_{2n} = S_{3n} - S_{2n} \Rightarrow 2\cdot S_{2n} = S_{3n} \Rightarrow 2\cdot\frac{2n^2 + 63n}{4} = \frac{9n^2 + 189n}{8} \; ;$$

$$4(2n^2 + 63n) = 9n^2 + 189n \quad ; \quad 8n^2 + 252n = 9n^2 + 189n \quad ; \quad 63\cancel{n} = n^{\cancel{2}}$$

$$\boxed{n = 63}$$

N11 P2 #12

$$(a_1-a_2)^2+(a_2-a_3)^2+\dots+(a_n-a_{n+1})^2=\frac{a_1^2(1-r)(1-r^{2n})}{1+r}$$

$\boxed{n=1}$ Debemos demostrar que: $(a_1-a_2)^2=\dfrac{a_1^2(1-r)(1-r^2)}{1+r}$.

$$\frac{a_1^2(1-r)(1-r^2)}{1+r}=\frac{a_1^2(1-r)\cancel{(1+r)}(1-r)}{\cancel{1+r}}=a_1^2(1-r)^2=[a_1(1-r)]^2=(a_1-a_1r)^2=(a_1-a_2)^2 \checkmark$$

$\boxed{n=k}$ Suponiendo cierto que: $(a_1-a_2)^2+\dots+(a_k-a_{k+1})^2=\dfrac{a_1^2(1-r)(1-r^{2k})}{1+r}$

debemos demostrar: $(a_1-a_2)^2+\dots+(a_k-a_{k+1})^2+(a_{k+1}-a_{k+2})^2=\dfrac{a_1^2(1-r)(1-r^{2(k+1)})}{1+r}$

$$(a_1-a_2)^2+\dots+(a_k-a_{k+1})^2+(a_{k+1}-a_{k+2})^2=\frac{a_1^2(1-r)(1-r^{2k})}{1+r}+(a_{k+1}-a_{k+2})^2=$$

$$=\frac{a_1^2(1-r)(1-r^{2k})}{1+r}+(a_1r^k-a_1r^{k+1})^2=\frac{a_1^2(1-r)(1-r^{2k})}{1+r}+[a_1r^k(1-r)]^2=$$

$$=\frac{a_1^2(1-r)(1-r^{2k})}{1+r}+a_1^2r^{2k}(1-r)^2=a_1^2(1-r)\cdot\left[\frac{1-r^{2k}}{1+r}+r^{2k}(1-r)\right]=$$

$$=a_1^2(1-r)\cdot\frac{1-r^{2k}+r^{2k}(1-r)(1+r)}{1+r}=\frac{a_1^2(1-r)}{1+r}\cdot\left(1-r^{2k}+r^{2k}(1-r^2)\right)=$$

$$=\frac{a_1^2(1-r)}{1+r}\cdot\left(1-\cancel{r^{2k}}+\cancel{r^{2k}}-r^{2k+2}\right)=\frac{a_1^2(1-r)(1-r^{2(k+1)})}{1+r} \checkmark$$

M12 P2 TZ2 #1

a) $S_{16}=212$ $\longrightarrow$ $S_{16}=\dfrac{a_1+a_{16}}{2}\cdot 16$

$a_5=8$

$212=\dfrac{a_1+a_1+15d}{2}\cdot 16$

$212=8(2a_1+15d)$

$53=2(2a_1+15d)$

$53=4a_1+30d$

$8=a_1+4d$

$$\begin{cases}4a_1+30d=53\\ a_1+4d=8\end{cases}$$

$4a_1+30d=53$

$-4a_1-16d=-32$

$14d=21$

$d=\dfrac{21}{14}=\boxed{\dfrac{3}{2}}$

$a_1+4\cdot\dfrac{3}{2}=8 \Rightarrow \boxed{a_1=2}$

b) $S_n>600$

$\dfrac{a_1+a_n}{2}\cdot n>600$; $\dfrac{a_1+a_1+(n-1)d}{2}\cdot n>600$; $\dfrac{2a_1+(n-1)d}{2}\cdot n>600$;

$\dfrac{4+(n-1)\cdot\frac{3}{2}}{2}\cdot n>600$; $\dfrac{8+3n-3}{4}\cdot n>600$; $(3n+5)\cdot n>2400$;

$3n^2+5n-2400>0$

$n=\dfrac{-5\pm\sqrt{25+28800}}{6}=$ 27'4... $\Rightarrow \boxed{n=28}$; -29'1...

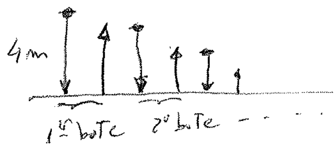
M12 P1 TZ1 #1

$$\sum_{r=1}^{\infty}k\cdot\left(\frac{1}{3}\right)^r=7$$

$\dfrac{k\cdot\frac{1}{3}}{1-\frac{1}{3}}=7$; $\dfrac{k}{3-1}=7$; $\boxed{k=14}$

M12 P2 TZ2 #8

a) h_n = altura que alcanza hecho el enésimo bote



$h_1 = 4 \cdot 0'95 = 3'8\ m$

$h_2 = 3'8 \cdot 0'95 = 3'61\ m$

$\boxed{h_n = 4 \cdot 0'95^n}$ $\quad h_4 = 4 \cdot 0'95^4 = \boxed{3'26\ m}$

b) $h_n < 1$

$4 \cdot 0'95^n < 1$

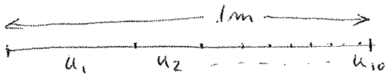
$4 \cdot 0'95^n = 1 \ ; \quad 0'95^n = 0'25 \ ; \quad n = \log_{0'95} 0'25 = 27'03 \Rightarrow \boxed{n = 28 \text{ botes}}$

c) $S_\infty = 4 + 3'8 + 3'61 + \cdots = \dfrac{4}{1-0'95} = \boxed{156\ m}$

N12 P2 #1

$102 + 105 + \cdots + 498 = \sum_{n=1}^{133} (3n+99) = \dfrac{102+498}{2} \cdot 133 = \boxed{39900}$

N12 P2 #5



P.G. $\quad u_1 = 8 \cdot u_{10}$, $\quad S_{10} = 1$

$\cancel{u_1} = 8 \cdot \cancel{u_1} \cdot r^9 \Rightarrow r^9 = \frac{1}{8} \ ; \quad r = \sqrt[9]{\frac{1}{8}} = \sqrt[9]{\frac{1}{2^3}} = \sqrt[3]{\frac{1}{2}}$

$S_{10} = \dfrac{u_1 \cdot r^{10} - u_1}{r-1}$

$1 = \dfrac{u_1 \cdot \left(\sqrt[3]{\frac{1}{2}}\right)^{10} - u_1}{\sqrt[3]{\frac{1}{2}} - 1} \Rightarrow u_1 = \dfrac{\sqrt[3]{\frac{1}{2}} - 1}{\sqrt[3]{\frac{1}{2^{10}}} - 1} \Rightarrow$

$\Rightarrow u_{10} = \frac{1}{8} u_1 = \frac{1}{8} \dfrac{\sqrt[3]{\frac{1}{2}} - 1}{\sqrt[3]{\frac{1}{2^{10}}} - 1} = 0'029 \ldots\ m \simeq \boxed{29\ mm}$

M13 P1 TZ1 #8

$\dfrac{1}{\log_2 x}, \dfrac{1}{\log_8 x}, \dfrac{1}{\log_{32} x}, \dfrac{1}{\log_{128} x}, \ldots$

$u_n = \dfrac{1}{\log_{2^{2n-1}} x}$

$S_{20} = 100 \Rightarrow \dfrac{\dfrac{1}{\log_2 x} + \dfrac{1}{\log_{2^{39}} x}}{2} \cdot 20 = 100 \ ; \quad \dfrac{1}{\log_2 x} + \dfrac{1}{\log_{2^{39}} x} = 10 \ ;$

$\dfrac{1}{\frac{\ln x}{\ln 2}} + \dfrac{1}{\frac{\ln x}{\ln 2^{39}}} = 10 \ ; \quad \dfrac{\ln 2}{\ln x} + \dfrac{\ln 2^{39}}{\ln x} = 10 \ ; \quad \ln 2 + 39 \ln 2 = 10 \ln x \ ;$

$40 \ln 2 = 10 \ln x \ ; \quad 4 \ln 2 = \ln x \ ; \quad \ln 2^4 = \ln x \Rightarrow \boxed{x = 16}$

M13
P1 TZ2
#6

P.G. $\left. \begin{array}{l} a_1 = a \\ S_\infty = 76 \end{array} \right| \Rightarrow \quad 76 = \dfrac{a}{1-r}$

P.G. $\left. \begin{array}{l} b_1 = a \\ \text{razón} = r^3 \\ S_\infty = 36 \end{array} \right| \Rightarrow \quad 36 = \dfrac{a}{1-r^3}$

; $\left. \begin{array}{l} 76(1-r) = a \\ 36(1-r^3) = a \end{array} \right\} \rightarrow 76 - 76r = 36 - 36r^3;$

$36r^3 - 76r + 40 = 0$

$9r^3 - 19r + 10 = 0$

$$\begin{array}{c|cccc} & 9 & 0 & -19 & 10 \\ 1 & & 9 & 9 & -10 \\ \hline & 9 & 9 & -10 & \boxed{0} \end{array}$$

$9r^2 + 9r - 10 = 0$

$r = \dfrac{-9 \pm \sqrt{81 + 360}}{18} = \dfrac{-9 \pm \sqrt{441}}{18} = \dfrac{-9 \pm 21}{18} \begin{array}{l} \nearrow \frac{12}{18} = \frac{2}{3} \\ \searrow -\frac{30}{18} = -\frac{5}{3} \end{array}$

$r = \begin{cases} \boxed{2/3} \\ \cancel{-5/3} \end{cases}$ → Única solución porque $r \in (-1, 1)$

M13
P2 TZ2
#5

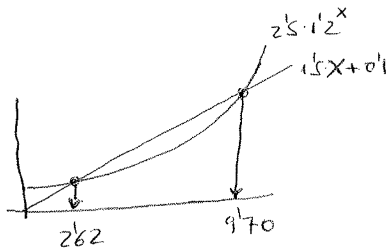
a) P.A. $\left. \begin{array}{l} u_1 = 1'6 \\ d = 1'5 \end{array} \right| \rightarrow u_n = 1'6 + 1'5(n-1) = 1'5n + 0'1$

P.G. $\left. \begin{array}{l} v_1 = 3 \\ r = 1'2 \end{array} \right| \rightarrow v_n = 3 \cdot 1'2^{n-1} = \dfrac{3}{1'2} 1'2^n = 2'5 \cdot 1'2^n$

$\rightarrow u_n - v_n = \boxed{1'5n + 0'1 - 2'5 \cdot 1'2^n}$

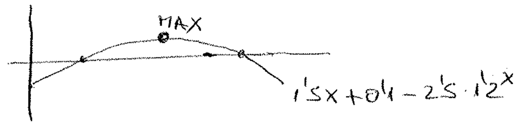
b) $u_n > v_n \Rightarrow 1'5n + 0'1 > 2'5 \cdot 1'2^n$

Hecho con calculadora gráfica:



$\boxed{n = 3, 4, 5, 6, 7, 8, 9}$

c) Hecho con calculadora gráfica:



Máximo: $(6'53 ; 1'67)$

La máxima diferencia estará entonces en $n=6$ ó en $n=7$:

$n = 6 \longrightarrow u_6 - v_6 = 1'6350$

$n = 7 \longrightarrow u_7 - v_7 = \boxed{1'642}$ para $n = 7$

M13
P2 TZ2
#11a

a) $S_n = \sum_{k=1}^{n} (2k - 1)$

b) $S_n = \dfrac{\cancel{1} + 2n - \cancel{1}}{2} \cdot n = \dfrac{2n^2}{2} = \boxed{n^2}$

c) $S_{47} - S_{14} = 47^2 - 14^2 = \boxed{2013}$

N13 P2 TZ2 #8

$5^{2n} - 24n - 1 = \dot{576}$

$\underline{\underline{n=1}}$ $\quad 5^2 - 24 \cdot 1 - 1 = 0 = \dot{576}$ ✓

$\underline{\underline{n=K}}$ $\quad$ Suponiendo cierto que $5^{2K} - 24K - 1$ es divisible entre 576

demostraremos que $5^{2(K+1)} - 24(K+1) - 1$ también lo es.

$5^{2(K+1)} - 24(K+1) - 1 = 5^{2K+2} - 24K - 24 - 1 = 5^{2K} \cdot 5^2 - 24K - 25 =$

$= 25 \cdot (5^{2K} - 24K - 1 + 24K + 1) - 24K - 25 = 25(5^{2K} - 24K - 1) + 600K + \cancel{25} - 24K - \cancel{25} =$

$= \underbrace{\underbrace{25(5^{2K} - 24K - 1)}_{\dot{576}} + \underbrace{576K}_{\dot{576}}}_{\dot{576}}$ ✓

N13 P1 #6

$n^3 + 11n = \dot{3}$

$\underline{\underline{n=1}}$ $\quad 1^3 + 11 \cdot 1 = 1 + 11 = 12 = \dot{3}$ ✓

$\underline{\underline{n=K}}$ $\quad$ Suponiendo cierto que $K^3 + 11K$ es divisible entre 3

demostraremos que $(K+1)^3 + 11(K+1)$ también lo es

$(K+1)^3 + 11(K+1) = K^3 + 3K^2 + 3K + 1 + 11K + 11 = (K^3 + 11K) + 3K^2 + 3K + 12 =$

$= \underbrace{\underbrace{(K^3 + 11K)}_{\dot{3}} + \underbrace{3(K^2 + K + 4)}_{\dot{3}}}_{\dot{3}}$ ✓

N13 P1 #7

a) P.G. $\quad \begin{cases} a_1 + a_2 = 10 \\ S_4 = 30 \end{cases} \quad \begin{cases} a_1 + a_1 r = 10 \\ \dfrac{a_1(r^4 - 1)}{r - 1} = 30 \end{cases} \longrightarrow a_1 = \dfrac{10}{1+r}$

$\dfrac{\frac{10}{1+r}(r^4 - 1)}{r-1} = 30 \quad ; \quad \dfrac{1\cancel{0}(r^4 - 1)}{r^2 - 1} = 3\cancel{0} \quad ; \quad r^4 - 1 = 3r^2 - 3 \quad ; \quad r^4 - 3r^2 + 2 = 0$

$r^2 = \dfrac{3 \pm \sqrt{9 - 8}}{2} = \dfrac{3 \pm 1}{2} \begin{cases} 2 \text{ ✓} \\ 1 \end{cases}$

b) $\underline{\underline{r = \sqrt{2}}} \longrightarrow a_1 = \boxed{\dfrac{10}{1+\sqrt{2}}}$

$S_{10} = \dfrac{a_1(r^{10} - 1)}{r - 1} = \dfrac{\frac{10}{1+\sqrt{2}}\left((\sqrt{2})^{10} - 1\right)}{\sqrt{2} - 1} = \dfrac{10(32 - 1)}{(\sqrt{2})^2 - 1} = \boxed{310}$

N13 P2 #2

a) P.A. $\quad \begin{cases} a_4 = 34 \\ a_{10} = 76 \end{cases} \quad \begin{cases} a_1 + 3d = 34 \\ a_1 + 9d = 76 \end{cases}$

$6d = 42 \quad ; \quad \boxed{d = 7} \longrightarrow \boxed{a_1 = 13} \longrightarrow a_n = 13 + 7(n-1) =$

$= 7n + 6$

b) $S_n > 5000$

$\dfrac{a_1 + a_n}{2} \cdot n > 500 \quad ; \quad \dfrac{13 + 7n + 6}{2} \cdot n > 5000 \quad ; \quad 7n^2 + 19n > 10000 \quad ;$

$7n^2 + 19n - 10000 > 0$

$n = \dfrac{-19 \pm \sqrt{361 + 280000}}{14} \begin{cases} 36'46\ldots \Rightarrow \boxed{n = 37} \\ -39'177\ldots \end{cases}$

Muestra 14
P2 #11

a)

$$S_1 = P + P\cdot\frac{I}{100} - R = P\left(1+\frac{I}{100}\right) - R$$

$$S_2 = S_1 + S_1\cdot\frac{I}{100} - R = S_1\cdot\left(1+\frac{I}{100}\right) - R = \left[P\left(1+\frac{I}{100}\right) - R\right]\left(1+\frac{I}{100}\right) - R =$$

$$= P\left(1+\frac{I}{100}\right)^2 - R\left(1+\frac{I}{100}\right) - R = P\left(1+\frac{I}{100}\right)^2 - R\left(1+\left(1+\frac{I}{100}\right)\right) ✓$$

$$S_n = P\left(1+\frac{I}{100}\right)^n - R\left(1+\left(1+\frac{I}{100}\right)+\left(1+\frac{I}{100}\right)^2+\cdots+\left(1+\frac{I}{100}\right)^{n-1}\right) =$$

$$= P\left(1+\frac{I}{100}\right)^n - R\,\frac{1\cdot\left(1+\frac{I}{100}\right)^n - 1}{\cancel{1}+\frac{I}{100}-\cancel{1}} =$$

$$= P\left(1+\frac{I}{100}\right)^n - \frac{100R}{I}\left(\left(1+\frac{I}{100}\right)^n - 1\right) ✓$$

b)

$P = \$5000$, $I = 1$, $n = 60$, $S_{60} = 0$

$$0 = 5000\cdot 1'01^{60} - \frac{100R}{1}\left(1'01^{60}-1\right)$$

$$R = \frac{5000\cdot 1'01^{60}}{100\left(1'01^{60}-1\right)} = \boxed{\$111'22}$$

$P = \$5000$, $I = 1$, $n = 20$, $R = \$111'22$

$$S_{20} = 5000\cdot 1'01^{20} - \frac{100\cdot 111'22}{1}\left(1'01^{20}-1\right) = \boxed{\$3652}$$

Muestra 14
P2 #2

P.G. $a_3 = 9$, $S_\infty = 64$

$$9 = a\cdot r^2$$

$$64 = \frac{a}{1-r} \rightarrow a = 64(1-r)$$

$$9 = 64(1-r)r^2$$

$$64r^3 - 64r^2 - 9 = 0$$

$$r = \boxed{0'75} \rightarrow a = \boxed{16}$$

(Hecho con calculadora gráfica)

Mayo 14
P2 TZ1
#7

$$7^{8n+3} + 2 = \dot{5}$$

$\underline{n=1}$ $\quad 7^{8\cdot 1+3} + 2 = 7^{11} + 2 = 1977326745 = \dot{5}$ ✓

$\underline{n=K}$ $\quad$ Suponiendo cierto que $7^{8K+3}+2$ es divisible entre 5 demostraremos que $7^{8(K+1)+3}+2$ también lo es.

$$7^{8(K+1)+3} + 2 = 7^{8K+8+3} + 2 = 7^8\cdot 7^{8K+3} + 2 =$$

$$= 7^8\cdot\left(7^{8K+3}+2-2\right) + 2 = 7^8\cdot\left(7^{8K+3}+2\right) - 7^8\cdot 2 + 2 =$$

$$= \underbrace{7^8\cdot\underbrace{\left(7^{8K+3}+2\right)}_{\dot{5}} - \underbrace{11529600}_{\dot{5}}}_{\dot{5}} ✓$$

M14
P2 TZ2
#1(a)

$$14 + 21 + 28 + \cdots 196$$

$$a_n = 7n + 7$$

$$a_n = 196 \Rightarrow 7n+7 = 196 \;;\; n = 27$$

$$S = \frac{14+196}{2}\cdot 27 = 2835$$

$$S_{27} = \sum_{n=1}^{27}(7n+7)$$

M14
P2 TZ2
#1(b)

P.A. $a_1 = 1000$, $d = -6$ $\rightarrow$ $a_n = 1000 + (-6)(n-1) = 1006 - 6n$

$S_n < 0$ $\rightarrow$ $\frac{1000 + 1006 - 6n}{2} \cdot n < 0$

$\frac{(2006 - 6n)\cdot n}{2} < 0$

$(1003 - 3n)\cdot n < 0 \Rightarrow 1003 - 3n < 0$

$n > \frac{1003}{3} = 334'3$

$\boxed{n = 335}$

N14
P1 #8

$(2n)! \geq 2^n \cdot (n!)^2$

$\underline{n=1}$ $(2\cdot 1)! = 2! = 2$ ✓
$2^1 \cdot (1!)^2 = 2 \cdot 1 = 2$

$\underline{n=k}$ Suponiendo cierto que $(2k)! \geq 2^k (k!)^2$
demostraremos que $\{2(k+1)\}! \geq 2^{k+1} \{(k+1)!\}^2$

$\{2(k+1)\}! = (2k+2)! = (2k+2)(2k+1)\cdot(2k)! \geq (2k+2)(2k+1)\cdot 2^k \cdot (k!)^2 =$

$= 2(k+1)\underbrace{(2k+1)}\cdot 2^k \cdot (k!)^2 \geq 2(k+1)(k+1)\, 2^k \cdot (k!)^2 = 2^{k+1}\cdot[(k+1)!]^2$ ✓

N14
P2 #7

a) P.A. $a_n \longrightarrow a_n = a + d(n-1)$ $(d \neq 0)$

P.G. $a_7, a_3, a_1 \longrightarrow a+6d,\ a+2d,\ a \Rightarrow \frac{a+2d}{a+6d} = \frac{a}{a+2d}$;

$\cancel{a^2} + 4ad + 4d^2 = \cancel{a^2} + 6ad$; $4d^2 = 2ad$; $2d = a \Rightarrow \boxed{d = \frac{a}{2}}$ ✓

b) $a_7 = 3 \rightarrow a + 6d = 3$; $a + 6\frac{a}{2} = 3$; $a + 3a = 3$; $a = \frac{3}{4} \rightarrow d = \frac{3}{8}$

P.A. $a_n = \frac{3}{4} + \frac{3}{8}(n-1)$ $\rightarrow a_7 = 3$, $\rightarrow a_3 = 1'5$, $\rightarrow a_1 = \frac{3}{4}$

$S_n = \frac{\frac{3}{4} + \frac{3}{4} + \frac{3}{8}(n-1)}{2} \cdot n =$

$= \boxed{\frac{(3n+11)n}{16}}$

P.G. $3, 1'5, \frac{3}{4}, \ldots$

$r = \frac{1}{2}$ $b_n = 3 \cdot \left(\frac{1}{2}\right)^{n-1}$

$S'_n = \frac{3(0'5^n - 1)}{0'5 - 1} = \boxed{-6(0'5^n - 1)}$

$S_n \geq S'_n + 200$

$\frac{3n^2 + 11n}{16} \geq -6(0'5^n - 1) + 200$

Hecho con calculadora gráfica: $n \geq 31'68 \Rightarrow \boxed{n = 32}$

M15 P1 TZ2 #12

a) $(x-\alpha)(x-\beta)(x-\gamma) = (x^2-(\alpha+\beta)x+\alpha\beta)(x-\gamma) =$

$= x^3-\gamma x^2-(\alpha+\beta)x^2+(\alpha+\beta)\gamma x+\alpha\beta x-\alpha\beta\gamma =$

$= x^3 \underbrace{-(\alpha+\beta+\gamma)}_{p\ \checkmark} x^2 + \underbrace{(\alpha\gamma+\beta\gamma+\alpha\beta)}_{q\ \checkmark} x \underbrace{-\alpha\beta\gamma}_{c\ \checkmark}$

b) $\left.\begin{array}{l} -6=-(\alpha+\beta+\gamma) \\ 18=\alpha\beta+\beta\gamma+\gamma\alpha \end{array}\right\}$; $\left.\begin{array}{l} \alpha+\beta+\gamma=6 \\ \alpha\beta+\beta\gamma+\gamma\alpha=18 \end{array}\right\}$

P.A. $\left.\begin{array}{l} \alpha=\beta-d \\ \gamma=\beta+d \end{array}\right| \to \beta-\cancel{d}+\beta+\beta+\cancel{d}=6$

$3\beta=6 \Rightarrow \boxed{\beta=2} \to \alpha=2-d$, $\gamma=2+d$

$18=2\alpha+2\gamma+\gamma\alpha$

$18=2(2-d)+2(2+d)+(2+d)(2-d)$;

$18=4-2\cancel{d}+4+2\cancel{d}+4-d^2$; $d^2=-6 \to c=-\alpha\beta\gamma=-(2-d)\cdot 2\cdot(2+d)=$

$=-2(4-d^2)=-2(4-(-6))=\boxed{-20}$

c) P.G. $\left.\begin{array}{l} \beta=\alpha r \\ \gamma=\alpha r^2 \end{array}\right|$ $\left.\begin{array}{l} \alpha+\alpha r+\alpha r^2=6 \\ \alpha^2 r+\alpha^2 r^3+\alpha^2 r^2=18 \end{array}\right\}$ $\left.\begin{array}{l} \alpha(1+r+r^2)=6 \\ \alpha^2 r(1+r^2+r)=18 \end{array}\right\} \to$

$\to \left.\begin{array}{l} 1+r+r^2=\frac{6}{\alpha} \\ \alpha^2 r(1+r+r^2)=18 \end{array}\right\} \to \alpha^{\cancel{2}} r\,\frac{6}{\cancel{\alpha}}=18$; $\alpha r=3 \Rightarrow \boxed{\beta=3} \to \alpha=3/r$, $\gamma=3r$

$\Rightarrow c=-\alpha\beta\gamma=-\frac{3}{\cancel{r}}\cdot 3\cdot 3\cancel{r}=\boxed{-27}$

M15 P1 TZ2 #13

a) $\frac{1}{\sqrt{n}+\sqrt{n+1}}=\frac{\sqrt{n}-\sqrt{n+1}}{(\sqrt{n}+\sqrt{n+1})(\sqrt{n}-\sqrt{n+1})}=\frac{\sqrt{n}-\sqrt{n+1}}{(\sqrt{n})^2-(\sqrt{n+1})^2}=\frac{\sqrt{n}-\sqrt{n+1}}{\cancel{n}-\cancel{n}-1}=\sqrt{n+1}-\sqrt{n}$ ✓

b) $n=1$ $\frac{1}{\sqrt{1}+\sqrt{2}}=\sqrt{2}-\sqrt{1} \Rightarrow \frac{1}{1+\sqrt{2}}=\sqrt{2}-1$

$\sqrt{2}-1=\frac{1}{1+\sqrt{2}}<\frac{1}{\sqrt{2}}$ ✓

al disminuir el denominador, crece la fracción

c) $\sum_{r=1}^{n}\frac{1}{\sqrt{r}}>\sqrt{n}$ $(n\geq 2)$

apartado (b)

$n=2$ $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}=1+\frac{1}{\sqrt{2}}>\cancel{1}+\sqrt{2}-\cancel{1}=\sqrt{2}$ ✓

$n=k$ Suponiendo cierto que $\sum_{r=1}^{k}\frac{1}{\sqrt{r}}>\sqrt{k}$

demostraremos que $\sum_{r=1}^{k+1}\frac{1}{\sqrt{r}}>\sqrt{k+1}$

$\sum_{r=1}^{k+1}\frac{1}{\sqrt{r}}=\sum_{r=1}^{k}\frac{1}{\sqrt{r}}+\frac{1}{\sqrt{k+1}}>\sqrt{k}+\frac{1}{\sqrt{k+1}}>$

Al aumentar el denominador, disminuye la fracción

$>\sqrt{k}+\frac{1}{\sqrt{k+1}+\sqrt{k}}=\cancel{\sqrt{k}}+\sqrt{k+1}-\cancel{\sqrt{k}}=$

apartado (a)

$=\sqrt{k+1}$ ✓