

1

Año 2001	Emisiones de CO ₂ (Miles de Tm)	Habitantes (miles de personas)	t.p.h	t.p.h ²
Alemania	870.762	82.537	10,55	111,30
Austria	69.120	8.067	8,57	73,41
Bélgica	126.803	10.356	12,24	149,93
Dinamarca	54.355	5.384	10,10	101,92
España	307.248	42.717	7,19	51,73
Finlandia	67.692	5.206	13,00	169,07
Francia	411.353	59.629	6,90	47,59
Grecia	105.875	11.018	9,61	92,34
Irlanda	46.460	3.964	11,72	137,37
Italia	460.760	57.321	8,04	64,61
Luxemburgo	5.482	448	12,24	149,73
Países Bajos	179.855	16.193	11,11	123,36
Portugal	64.892	10.408	6,23	38,87
Reino Unido	560.849	59.329	9,45	89,36
Suecia	55.269	8.941	6,18	38,21
			143,13	1438,83

$$\bar{X} = \frac{14313}{15} = \boxed{9542} \quad \sigma = \sqrt{\frac{143883}{15} - 9542^2} = \boxed{2207}$$

España estaba en el 2001 muy por debajo de la media europea, ni siquiera está en el intervalo $(\bar{x} - \sigma, \bar{x} + \sigma)$ al que pertenece la mayoría de los países

2

$$a < b < c$$

$$\text{mediana} = 11 \rightarrow \boxed{b = 11}$$

$$\text{media} = 9 \rightarrow \frac{a+11+c}{3} = 9 \quad \left\{ \begin{array}{l} a+c = 16 \\ c-a = 10 \end{array} \right. \Rightarrow \boxed{\begin{array}{l} a=3 \\ c=13 \end{array}}$$

$$\text{rango} = 10 \rightarrow c - a = 10$$

3

Buscando 20, 40 y 60 \rightarrow $\begin{array}{l} Q_1 = 130 \text{ ag} \\ Q_2 = 135 \text{ ag} \\ Q_3 = 141 \text{ ag} \end{array} \rightarrow \boxed{\text{mediana} = 135 \text{ ag}} \rightarrow \text{rango intercuartil} = 141 - 130 = \boxed{11 \text{ ag}}$

4

X	P
6	4/40
4	12/40
2	24/40

$$\mu = 6 \cdot \frac{4}{40} + 4 \cdot \frac{12}{40} + (-3) \cdot \frac{24}{40} = \frac{0}{40} = \boxed{0}$$

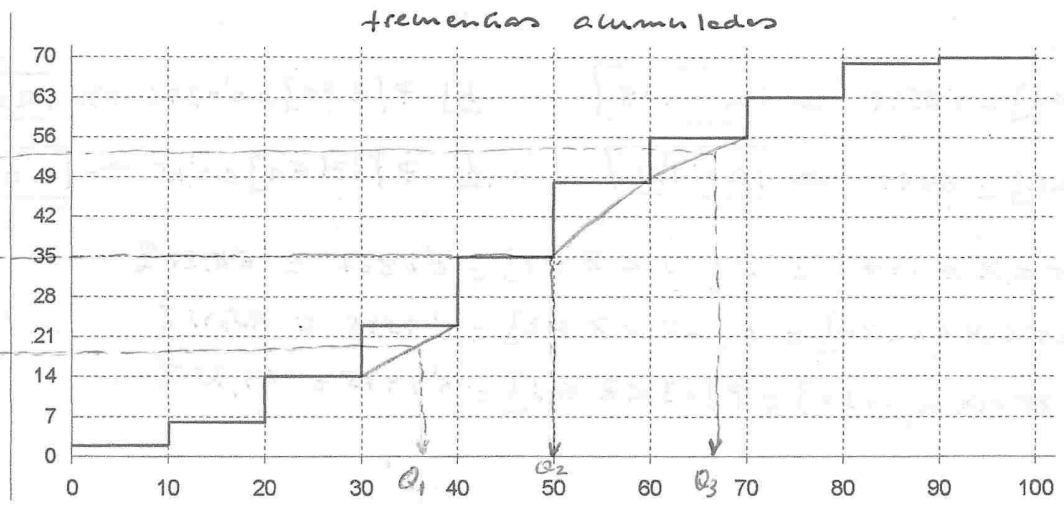
Es un juego equilibrado, jugando muchas partidas, compensamos ganancias con pérdidas.

5) a) mode $\in [50,60) \Rightarrow$ mode = 55h

Las frecuencias acumuladas son:

t	[0,10)	[10,20)	[20,30)	[30,40)	[40,50)	[50,60)	[60,70)	[70,80)	[80,90)	[90,100)
N	2	4	8	9	12	13	8	7	6	1
F.A.	2	6	14	23	35	48	56	63	69	70

Buscamos $\frac{3 \cdot 70}{4} = 52.5$
 $\frac{2 \cdot 70}{4} = 35$
 $\frac{70}{4} = 17.5$



$Q_1 \approx 35h$
 $Q_2 \approx 50h$
 $Q_3 \approx 65h$

Interpolando se obtiene una mayor aproximación:

$Q_1 = 33.8$
 $Q_2 = 50$
 $Q_3 = 65.625$

b) $\bar{X} = 49.86$ $\sigma = 21.43$ $\sigma^2 = 21.59$

6)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

x	p	xP	x ² P
0	6/36	0	0
1	10/36	10/36	10/36
2	8/36	16/36	32/36
3	6/36	18/36	54/36
4	4/36	16/36	64/36
5	2/36	10/36	50/36
	70/36	210/36	

$\mu = 0 \cdot \frac{6}{36} + 1 \cdot \frac{10}{36} + 2 \cdot \frac{8}{36} + 3 \cdot \frac{6}{36} + 4 \cdot \frac{4}{36} + 5 \cdot \frac{2}{36} = \frac{35}{18}$

$\sigma^2 = \sqrt{\frac{210}{36} - \left(\frac{35}{18}\right)^2} = \sqrt{\frac{665}{324}} = 1.433$

7)

$B(15, 0.1)$
 $P[3 \leq X < 6] = P[X=3] + P[X=4] + P[X=5] = \binom{15}{3} 0.1^3 \cdot 0.9^{12} + \binom{15}{4} 0.1^4 \cdot 0.9^{11} + \binom{15}{5} 0.1^5 \cdot 0.9^{10} = 0.182$

8)

6H | 4M | X = n° mujeres seleccionadas.

$P[X=0] = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{120}{720} = \frac{5}{30}$
 $P[X=1] = 3 \cdot \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{360}{720} = \frac{15}{30}$
 $P[X=2] = 3 \cdot \frac{6}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{216}{720} = \frac{9}{30}$
 $P[X=3] = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{24}{720} = \frac{1}{30}$

x	p	xP	x ² P
0	5/30	0	0
1	15/30	15/30	15/30
2	9/30	18/30	36/30
3	1/30	3/30	9/30
	36/30	60/30	

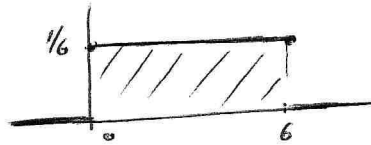
$\mu = \frac{36}{30} = 1.2$ $\sigma^2 = \frac{60}{30} - 1.2^2 = 0.56$

9) $P[X=k] = e^{-\lambda} \frac{\lambda^k}{k!}$ ($k=0, 1, 2, \dots$)

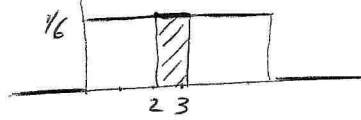
$P[X > 2] = 1 - P[X=0] - P[X=1] - P[X=2] = 1 - e^{-11} \cdot \frac{11^0}{0!} - e^{-11} \cdot \frac{11^1}{1!} - e^{-11} \cdot \frac{11^2}{2!} =$
 $= 1 - e^{-11} \left(1 + 11 + \frac{11^2}{2} \right) = 1 - 0.9004 = 0.0999 \approx \boxed{0.1}$

10) a) $f(x) \geq 0 \quad \forall x \in \mathbb{R}$ ✓

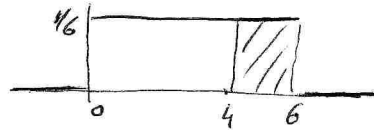
Area = $\frac{1}{6} \times 6 = 1$ ✓



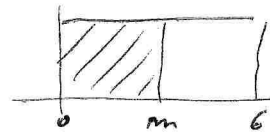
b) $P[2 \leq X \leq 3] = \frac{1}{6} \cdot 1 = \boxed{\frac{1}{6}}$



c) $P[X > 4] = \frac{1}{6} \cdot 2 + 0 = \boxed{\frac{1}{3}}$

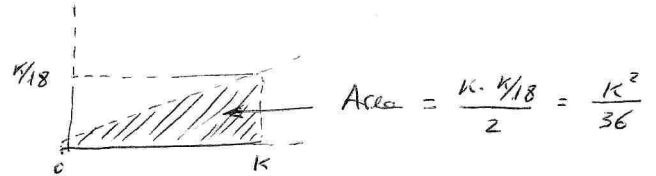
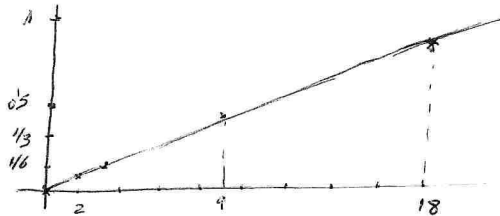


d) m debe cumplir: $P[X \leq m] = 0.5$
 obviamente es $\boxed{m=3}$

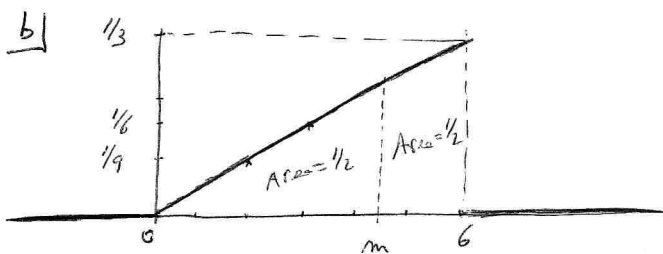


11) $f(x) = \begin{cases} \frac{x}{18} & \text{si } 0 \leq x \leq K \\ 0 & \text{otro caso} \end{cases}$

x	f
0	0
2	1/9
3	1/6
4	1/2
18	1
K	1/18



a) Area = 1 $\Rightarrow \frac{K^2}{36} = 1$; $K^2 = 36$; $\boxed{K=6}$

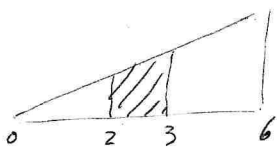


x	f
0	0
2	1/9
3	1/6
6	1/3

La mediana m, separa el área total en dos mitades.

$\frac{m \cdot \frac{m}{18}}{2} = \frac{1}{2}$; $\frac{m^2}{36} = \frac{1}{2}$; $m^2 = 18$ mediana = $\sqrt{18}$

c) $\text{Area} = \frac{\frac{2}{18} + \frac{3}{18}}{2} \times 1 = \frac{5}{36}$



$P[2 \leq X \leq 3] = \frac{5}{36}$

⑫ a) $P[Z \leq 0.43] = 0.6664$ b) $P[Z \leq -1.46] = 0.0721$ c) $P[Z > 1.61] = 0.0537$
 d) $P[Z > -2.06] = 0.9803$ e) $P[0.91 < Z \leq 2.3] = 0.1707$ f) $P[-1.72 < Z \leq -0.23] = 0.3663$
 g) $P[-0.74 < Z \leq 1.5] = 0.7035$

13 a) $P[Z \leq a] = 0.8599 \Rightarrow a = 1.08$ b) $P[Z \leq a] = 0.0392 \Rightarrow a = -1.76$

c) $P[Z > a] = 0.951 \Rightarrow a = 1.31$

d) $P[|Z| \leq a] = 0.75 \rightarrow P[-a \leq Z \leq a] = 0.75 \rightarrow P[Z \leq a] - [1 - P[Z \leq a]] = 0.75 \rightarrow$
 $2P[Z \leq a] = 1.75 \rightarrow P[Z \leq a] = 0.875 \Rightarrow a = 1.15$

14 $P[-a < Z \leq a] = P[Z \leq a] - (1 - P[Z \leq a]) = 2 \cdot P[Z \leq a] - 1$



$N(\mu, \sigma)$

a) $P[\mu - \sigma < X \leq \mu + \sigma] = P[-\sigma < X - \mu \leq \sigma] = P[-1 < \frac{X - \mu}{\sigma} \leq 1] = P[-1 < Z \leq 1] =$
 $= 0.6827 = 68.27\%$

b) $P[\mu - 2\sigma < X \leq \mu + 2\sigma] = P[-2 < Z \leq 2] = 0.9545 = 95.45\%$

c) $P[\mu - 3\sigma < X \leq \mu + 3\sigma] = P[-3 < Z \leq 3] = 0.9975 = 99.75\%$

15 $N(66, 5)$

$P[65 < X \leq 70] = P[-0.2 < Z \leq 0.8] = 0.3674$

$0.3674 \cdot 800 = 293.92 \approx 294 \text{ minutos}$

16 $N(5, 1.5)$

$P[X > a] = 0.10 \rightarrow P[Z > \frac{a-5}{1.5}] = 0.10 \Rightarrow \frac{a-5}{1.5} = 1.285 \Rightarrow a = 6.9275$

17) $N(12, 4)$

$P[X > 15] = P[Z > \frac{3}{4}] = 0.2266 \Rightarrow 0.2266 \cdot 50 = \boxed{11.33 \text{ muestras tóxicas}}$

18) $N(L, 0.12)$

$P[X > 0.7] = 20\% = 0.2 \Rightarrow P[Z > \frac{0.7-L}{0.12}] = 0.2 \Rightarrow \frac{0.7-L}{0.12} = 0.8416 \Rightarrow$

$\Rightarrow L = 0.7 - 0.12 \cdot 0.8416 = \boxed{0.599 \text{ metros}}$

19)

M = Llegar a tiempo los materiales

T = Acabar la obra a tiempo

	M	\bar{M}	
T	0.55	0.65	0.60
\bar{T}	0.30	0.10	0.40
	0.85	0.15	1

a) $P(M \cap T) = 0.55$

$P(M) \cdot P(T) = 0.85 \cdot 0.60 = 0.51$

M, T son dependientes

b) $P(\bar{T}/M) = \frac{P(\bar{T} \cap M)}{P(M)} = \frac{0.30}{0.85} = \frac{30}{85} = \boxed{0.3529}$

c)
$$P = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{2}{8} \cdot \frac{5}{7} \cdot \frac{4}{6} \cdot \frac{5!}{2!2!} = \boxed{0.2381}$$

d) $N(42, 4)$

$P[X \geq 48] = 10\% = 0.1 \Rightarrow P[Z \geq \frac{6}{4}] = 0.1 \Rightarrow \frac{6}{4} = 1.281 \Rightarrow \boxed{\sigma = 4.68}$

$P[X > 40] = P[Z > \frac{-2}{4.68}] = \boxed{0.6654}$

$P[X > 40 \wedge X > 40] = 0.6654^2 = \boxed{0.4428}$

20) $N(\mu, \sigma)$

$P[X > 50.32] = 0.119 \Rightarrow P[Z > \frac{50.32 - \mu}{\sigma}] = 0.119 \Rightarrow \frac{50.32 - \mu}{\sigma} = 1.18$

$P[X < 43.56] = 0.305 \Rightarrow P[Z < \frac{43.56 - \mu}{\sigma}] = 0.305 \Rightarrow \frac{43.56 - \mu}{\sigma} = -0.51$

$$\Rightarrow \begin{cases} 50.32 - \mu = 1.18\sigma \\ 43.56 - \mu = -0.51\sigma \end{cases} \Rightarrow \frac{6.76}{1.69\sigma} = 1.69\sigma \Rightarrow \boxed{\sigma = 4} \Rightarrow \mu = 50.32 - 1.18 \cdot 4 = \boxed{45.6}$$

b) $P[|X - \mu| < 5] = P[-5 < X - 45.6 < 5] = P[-\frac{5}{4} < Z < \frac{5}{4}] = \boxed{0.7887}$