Integrales inmediatas:

Calcula:

1.- $\int x^2 dx =$ $\frac{x^3}{3} + c$

2.- $\int x^{-4} dx =$ $\frac{x^{-4+1}}{-4+1} + c = \frac{x^{-3}}{-3} + c = \frac{-1}{x^3} + c$

3.- $\int -4x^3 dx =$ $-4 \int x^3 \cdot dx = \cancel{-4} \frac{x^4}{\cancel{4}} + c = -x^4 + c$

4.- $\int \frac{3}{4} x^{-2} dx =$ $\frac{3}{4} \int x^{-2} \cdot dx = \frac{\cancel{3}}{4} \frac{x^{-3}}{\cancel{-3}} + c = \cancel{3} \frac{-1}{4x^3} + c$

5.- $\int 3e^x dx =$ $3 \cdot \int e^x dx = 3 \cdot e^x + c$

6.- $\int 4(x^3 - 2x^2 + 23)(3x^2 - 4x) dx =$ $4 \int u^1 \cdot u' \, du = \frac{4 \cdot (x^3 - 2x^2 + 23)^2}{2} + c = 2(x^3 - 2x^2 + 23)^2 + c$

7.- $\int 3 dx =$ $3x + c$

8.- $\int \frac{1}{x} dx =$ $\ln|x| + c$

9.- $\int \frac{3}{x} dx =$ $3 \int \frac{1}{x} dx = 3 \int \frac{dx}{x} = 3 \cdot \ln|x| + c$

10.- Halla las primitivas de $f(x) = 5x$ $\longrightarrow F(x) = \int 5x \cdot dx = \frac{5x^2}{2} + c$

11.- Halla la primitiva de $f(x) = 3x^2$ que en x=1 vale 5.

$F(x) = \int 3x^2 \cdot dx = \frac{3x^3}{3} + c$

Si $F(1) = 5 \Rightarrow \frac{3 \cdot 1^3}{3} + c = 5 \Rightarrow c = 4$

luego $\boxed{F(x) = x^3 + 4}$

Ⓑ

$e^{-x} \xrightarrow{d} e^{-x}(-1)$

Ejercicios de integrales inmediatas (II):

1ª) $\int (e^x + e^{-x})dx =$ $\int e^x \cdot dx + \int e^{-x} \cdot dx = e^x - e^{-x} + c$

orientación: descomponer en dos sumandos

2ª) $\int \frac{dx}{(x-1)^2} =$ $\int (x-1)^{-2} \cdot dx = \frac{(x-1)^{-1}}{-1} + c = \frac{-1}{x-1} + c$

Pasar la potencia al numerador

3ª) $\int \frac{5x}{\sqrt{1+x^2}} dx =$ $\int 5x \cdot (1+x^2)^{-1/2} \cdot dx = 5 \cdot \frac{1}{2} \int 2x(1+x^2)^{-1/2} \cdot dx = \frac{5}{2} \cdot \frac{(1+x^2)^{1/2}}{1/2} + c$ $= \frac{5}{\cancel{2}} \cdot \sqrt{1+x^2} + c$

Como raíz. O pasar la raíz a potencia y ponerla en el numerador.

4ª) $\int x\sqrt{1+x^2}\,dx =$ $\frac{1}{2} \int 2x \cdot (1+x^2)^{1/2} \cdot dx = \frac{1}{2} \cdot \frac{(1+x^2)^{3/2}}{\frac{3}{2}} + c = \frac{\cancel{2}}{\cancel{2}} \cdot \frac{\sqrt{(1+x^2)^3}}{3} + c$

Pasar la raíz a potencia, o como raíz.

5ª) $\int x^2(x+x^2)dx =$ $\int (x^3 + x^4)\,dx = \frac{x^4}{4} + \frac{x^5}{5} + c$

Efectuar la multiplicación antes

6ª) $\int (x-1)(x+1)dx =$ $\int (x^2 - 1)\,dx = \frac{x^3}{3} - x + c$

Efectuar la multiplicación

7ª) $\int (4x^5 - 3x^4 + 2x + 1)dx =$ $4\frac{x^6}{6} - 3\frac{x^5}{5} + 2\frac{x^2}{2} + x + c = \frac{2}{3}x^6 - \frac{3}{5}x^5 + x^2 + x + c$

Descomponer

8ª) $\int (3 + \cos x)dx =$ $\int 3\,dx + \int \cos x \cdot dx = 3x + \operatorname{sen} x + c$

Descomponer

9ª) $\int (x+1)^2 dx =$ $\frac{(x+1)^3}{3} + c$

potencia de una función

10ª) $\int (2x+1)(x^2+x+1)^{20} dx =$ $\frac{(x^2+x+1)^{21}}{21} + c$

Potencia de una función

11ª) $\int (x^5 + 3x^3)dx =$ $\frac{x^6}{6} + 3\frac{x^4}{4} + c$

Descomponer

12ª) $\int (x^2 + 2x)(x^3 + x)dx =$ $\int (x^5 + x^3 + 2x^4 + 2x^2)\,dx = \frac{x^6}{6} + \frac{x^4}{4} + 2\frac{x^5}{5} + 2\frac{x^3}{3} + c$

Efectuar la multiplicación

C

EJERCICIOS DE INTEGRALES

A

Halle la antiderivada de cada función.

1 x^7 **2** x^4 **3** x^{-2} **4** $x^{-\frac{1}{2}}$

5 $x^{\frac{1}{3}}$ **6** $x^{\frac{2}{3}}$ **7** $\frac{1}{x^4}$ **8** $\frac{1}{x^{12}}$

9 $\sqrt[3]{x}$ **10** $\sqrt[2]{x^3}$ **11** $\frac{1}{\sqrt[5]{x}}$ **12** $\frac{1}{\sqrt[3]{x^2}}$

B

1 $\int x^3\,dx$ **2** $\int \frac{1}{t^2}\,dt$

3 $\int \sqrt[3]{x^4}\,dx$ **4** $\int 2\,du$

5 $\int (3x^2 + 2x + 1)\,dx$ **6** $\int \frac{4}{x^3}\,dx$

7 $\int (t^2 + \sqrt[4]{t})\,dt$ **8** $\int (\sqrt[3]{x^2} + 1)\,dx$

9 $\int (5x^4 + 12x^3 + 6x - 2)\,dx$ **10** $\int dt$

11 Sea $f(x) = x^3 + \frac{4}{x^2}$.
Halle: **a** $f'(x)$ **b** $\int f(x)\,dx$

12 Sea $g(x) = 30\sqrt[3]{x}$.
Halle: **a** $g'(x)$ **b** $\int g(x)\,dx$

C

Halle la integral indefinida.

1 $\int \frac{2}{x}\,dx$ **2** $\int 3e^{x}\,dx$

3 $\int \frac{1}{4t}\,dt$ **4** $\int e^{\ln x}\,dx$

5 $\int (2x+3)^2\,dx$ **6** $\int \frac{2x^3 + 6x^2 + 5}{x}\,dx$

7 $\int \ln e^{u^2}\,du$ **8** $\int (x-1)^3\,dx$

9 $\int \frac{e^x + 1}{2}\,dx$ **10** $\int \frac{x^2 + x + 1}{\sqrt{x}}\,dx$

D

D

Halle la integral indefinida en las preguntas 1 a 10.

1 $\int (2x+5)^2\,dx$ **2** $\int (-3x+5)^3\,dx$ **3** $\int e^{\frac{1}{2}x-3}\,dx$

4 $\int \frac{1}{5x+4}\,dx$ **5** $\int \frac{3}{7-2x}\,dx$ **6** $\int 4e^{2x+1}\,dx$

7 $\int 6(4x-3)^7\,dx$ **8** $\int (7x+2)^{\frac{1}{2}}\,dx$ **9** $\int \left(e^{4x}+\frac{4}{3x-5}\right)dx$

10 $\int \frac{2}{3(4x-5)^3}\,dx$

E

1 $\int (2x^2+5)^2(4x)\,dx$ **2** $\int \frac{3x^2+2}{x^3+2x}\,dx$

3 $\int (6x+5)\sqrt{3x^2+5x}\,dx$ **4** $\int 4x^3e^{x^4}\,dx$

5 $\int \frac{2x+3}{(x^2+3x+1)^2}\,dx$ **6** $\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

7 $\int x^2(2x^3+5)^4\,dx$ **8** $\int \frac{2x+1}{\sqrt[4]{x^2+x}}\,dx$

9 $\int (8x^3-4x)(x^4-x^2)^3\,dx$ **10** $\int \frac{4-3x^2}{x^3-4x}\,dx$

Integrales en problemas de BI desde 2014:

1. Find $\int \sin 3x \cos 3x\,dx$.

2. Let $f(x)=x^2$.

   Find $\int_1^2 \left(f(x)\right)^2 dx$.

3 Let $\int_{\pi}^{a} \cos 2x\,dx = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

4 Sabiendo que $h'(x)=4\cos 2x$, halle $h(x)$.

5 Halle $\int \frac{2x}{x^2+5}\,dx$.

6 Integral de:
$f(x)=5-x^2$.

E

7	Integral de $f(x)=\dfrac{x}{x^2+1}$
8	Integral de $f(x)=-x^4+2x^3-1$
9	Sea $g(x)=\dfrac{\ln x}{x}$. (a) Halle $g'(x)$. (b) Halle $\int g(x)\,dx$.
10	Integrales de $f(x)=\dfrac{9}{x+2}$ y $g(x)=3x^2$,
11	Let $f'(x)=6x^2-5$. Given that $f(2)=-3$, find $f(x)$.
12	Integral de $f(x)=2\ln(x-3)$
13	Integrales de $f(x)=x^2$ and $g(x)=3\ln(x+1)$
14	Integrales de $f(x)=xe^{-x}$ y $g(x)=-3f(x)+1$.
15	(a) Find $\int xe^{x^2-1}dx$. (b) Find $f(x)$, given that $f'(x)=xe^{x^2-1}$ and $f(-1)=3$.
16	Sea $f'(x)=\dfrac{3x^2}{(x^3+1)^5}$. Sabiendo que $f(0)=1$, halle $f(x)$.
17	Integrales de $f(x)=\ln x$ and $g(x)=3+\ln\left(\dfrac{x}{2}\right)$.
18	Integral de $f(x)=-0{,}5x^4+3x^2+2x$.

Ⓕ

Integrales en exámenes EBAU y PAU Asturias

Junio 2017

2. Dada la función $f(x) = x^3 - 3x^2 + 2x$, se pide:

1 a) **[0,75 puntos]** Encontrar la primitiva F de f verificando que $F(2) = 1$.

Julio 2017

2. Si x representa el volumen de producción de una fábrica, el coste marginal de la misma viene dado por la función $f(x) = 5 + 6x + 24x^2$. Se pide:

2 a) **[0,75 puntos]** Encontrar la función del coste total F, si se sabe que dicha función viene dada por la primitiva F de f que verifica que $F(2) = 90$.

2016

3. Dada la función $f(x) = 4x - x^3$, se pide:

3 a) Encontrar la primitiva F de f verificando que $F(2) = 7$.

2016

2. Dada la función $f(x) = x^3 - 5x^2 + 4x$, se pide:

4 a) Encontrar una primitiva F de f verificando que $F(2) = 1$.

2016

3. La propensión marginal al consumo viene dada por una función f con $f(x) = 0'6 - 0'01x$, donde x representa los ingresos. Se pide:

a) Encontrar la función de consumo F, si se sabe que dicha función viene dada por la primitiva F de f que verifica que $F(0) = 0'2$.

5

2016

2. La función de costes marginales de una empresa es $f(x) = \frac{20}{(x+2)^2}$, se pide:

6 a) Encontrar la primitiva F de f verificando que $F(3) = 0$.

2015

2. La función de costes marginales de una empresa es $f(x) = \frac{10}{(x+1)^2}$, se pide:

7 a) Encontrar la primitiva F de f verificando que $F(4) = 0$.

2015

2. Si x representa el volumen de producción de una fábrica, el coste marginal de la misma viene dado por la función $f(x) = 3 + 8x + 15x^2$. Se pide:

8 a) Encontrar la función del coste total F, si se sabe que dicha función viene dada por la primitiva F de f que verifica que $F(0) = 100$.

Ejercicios de integrales   Soluciones.

(A) Halle la antiderivada de cada función

(1) $\int x^7 \cdot dx = \frac{x^8}{8} + c$

(2) $\int x^4 \cdot dx = \frac{x^5}{5} + c$

(3) $\int x^{-2}\, dx = \frac{x^{-1}}{-1} + c = \frac{-1}{x} + c$

(4) $\int x^{-1/2} \cdot dx = \frac{x^{1/2}}{1/2} + c = 2\sqrt{x} + c$

(5) $\int x^{1/3}\, dx = \frac{x^{4/3}}{4/3} + c = \frac{3}{4} \cdot \sqrt[3]{x^4} + c$

(6) $\int x^{2/5}\, dx = \frac{x^{7/5}}{7/5} + c = \frac{5}{7} \cdot \sqrt[5]{x^7} + c = \frac{5}{7} \cdot x \cdot \sqrt[5]{x^2} + c$

(7) $\int \frac{1}{x^4} \cdot dx = \int x^{-4} \cdot dx = \frac{x^{-3}}{-3} + c = \frac{-1}{3x^3} + c$

(8) $\int \frac{1}{x^{12}} \cdot dx = \int x^{-12} \cdot dx = \frac{x^{-11}}{-11} + c = \frac{-1}{11 \cdot x^{11}} + c$

(9) $\int \sqrt[3]{x} \cdot dx = \int x^{1/3} \cdot dx = \frac{x^{4/3}}{4/3} + c = \frac{3}{4} \cdot \sqrt[3]{x^4} + c = \frac{3}{4} \cdot x \cdot \sqrt[3]{x} + c$

(10) $\int \sqrt[7]{x^3} \cdot dx = \int x^{3/7} \cdot dx = \frac{x^{10/7}}{10/7} + c = \frac{7}{10} \cdot \sqrt[7]{x^{10}} + c = \frac{7}{10} \cdot x \sqrt[7]{x^3} + c$

(11) $\int \frac{1}{\sqrt[5]{x}} \cdot dx = \int x^{-1/5} \cdot dx = \frac{x^{4/5}}{4/5} + c = \frac{5}{4} \cdot \sqrt[5]{x^4} + c$

(12) $\int \frac{1}{\sqrt[3]{x^2}} \cdot dx = \int x^{-2/3} \cdot dx = \frac{x^{1/3}}{1/3} + c = 3 \cdot \sqrt[3]{x} + c$

(B)

(1) $\int x^3 \, dx = \frac{x^4}{4} + c$

(2) $\int \frac{1}{t^2} \cdot dt = \int t^{-2} \cdot dt = \frac{t^{-1}}{-1} + c = \frac{-1}{t} + c$

(3) $\int \sqrt[5]{x^4} \cdot dx = \int x^{4/5} \cdot dx = \frac{x^{9/5}}{\frac{9}{5}} + c = \frac{5}{9} \cdot \sqrt[5]{x^9} + c = \frac{5}{9} \cdot x \cdot \sqrt[5]{x^4} + c$

(4) $\int 2 \cdot du = 2u + c$

(5) $\int (3x^2 + 2x + 1) \, dx = \cancel{3} \frac{x^3}{\cancel{3}} + \cancel{2} \frac{x^2}{\cancel{2}} + x + c$

(6) $\int \frac{4}{x^3} \cdot dx = \int 4 \cdot x^{-3} dx = 4 \cdot \frac{x^{-2}}{-2} + c = \frac{-2}{x^2} + c$

(7) $\int (t^2 + \sqrt[4]{t}) \, dt = \int (t^2 + t^{1/4}) \, dt = \frac{t^3}{3} + \frac{t^{5/4}}{\frac{5}{4}} + c = \frac{t^3}{3} + \frac{4}{5} \sqrt[4]{t^5} + c$

$= \frac{t^3}{3} + \frac{4}{5} t \cdot \sqrt[4]{t} + c$

(8) $\int (\sqrt[3]{x^2} + 1) \, dx = \int (x^{2/3} + 1) \, dx = \frac{x^{5/3}}{\frac{5}{3}} + x = \frac{3}{5} \sqrt[3]{x^5} + x + c =$

$= \frac{3}{5} \cdot x \sqrt[3]{x^2} + x + c$

(9) $\int (5x^4 + 12x^3 + 6x - 2) \, dx = 5 \frac{x^5}{5} + 12 \frac{x^4}{4} + 6 \frac{x^2}{2} - 2x + c =$

$= x^5 + 3x^4 + 3x^2 - 2x + c$

(10) $\int dt = t + c$

11 a/

$$f(x) = x^3 + \frac{4}{x^2}$$

$$f'(x) = 3x^2 + \frac{-2x \cdot 4}{x^4} = 3x^2 - \frac{8x}{x^4} = 3x^2 - \frac{8}{x^3}$$

b/

$$\int f(x) = \int \left(x^3 + \frac{4}{x^2}\right) dx = \int (x^3 + 4x^{-2})\, dx = \frac{x^4}{4} + 4 \cdot \frac{x^{-1}}{-1} + c$$

$$= \frac{x^4}{4} - \frac{4}{x} + c$$

12

$$g(x) = 30\sqrt[5]{x} = 30 \cdot x^{1/5}$$

a/

$$g'(x) = 30 \cdot \frac{1}{5} \cdot x^{-4/5} = \frac{6}{x^{4/5}} = \frac{6}{\sqrt[5]{x^4}} = \frac{6}{x\sqrt[5]{x}}$$

b/

$$\int (30 \cdot x^{1/5})\, dx = 30 \cdot \frac{x^{6/5}}{6/5} = \frac{5 \cdot 30}{6} \cdot \sqrt[5]{x^6} + c = 25 \cdot x\sqrt[5]{x} + c$$

Ⓒ Halle la integral indefinida de:

① $\int \frac{2}{x} \cdot dx = 2\int \frac{1}{x} \cdot dx = 2 \ln|x| + c$

② $\int 3 \cdot e^x \cdot dx = 3\int e^x \cdot dx = 3 \cdot e^x + C$

③ $\int \frac{1}{4t}\, dt = \frac{1}{4}\int \frac{1}{t}\, dt = \frac{1}{4} \cdot \ln|t| + c$

④ $\int e^{\ln x} \cdot dx = \int x \cdot dx = \frac{x^2}{2} + c$; Nota $e^{\ln x} \Rightarrow x$

⑤ $\frac{1}{2}\int 2(2x+3)^2 \cdot dx = \frac{1}{2} \frac{(2x+3)^3}{3} + c = \frac{(2x+3)^3}{6} + c$

⑥ $\int \frac{2x^3 + 6x^2 + 5}{x} \cdot dx = \int \left(2x^2 + 6x + \frac{5}{x}\right) dx = 2\frac{x^3}{3} + \frac{6x^2}{2} + 5 \ln|x| + c$

⑦ $\int \ln e^{u^2} \cdot du = \int u^2 \cdot du = \frac{u^3}{3} + c$; Nota $\ln e^{u^2} \Rightarrow u^2$

⑧ $\int (x-1)^3 dx = \frac{(x-1)^4}{4} + c$

⑨ $\int \frac{e^x + 1}{2} \cdot dx = \int \left(\frac{e^x}{2} + \frac{1}{2}\right) dx = \frac{1}{2}\int e^x \cdot dx + \int \frac{1}{2}\, dx = \frac{1}{2} e^x + \frac{1}{2} x + c$

⑩ $\int \frac{x^2 + x + 1}{\sqrt{x}} \cdot dx = \int \frac{x^2 + x + 1}{x^{1/2}}\, dx = \int x^{3/2} \cdot dx + \int x^{1/2} \cdot dx + \int x^{-1/2} \cdot dx =$

$= \frac{x^{5/2}}{\frac{5}{2}} + \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + c = \frac{2}{5}\sqrt{x^5} + \frac{2}{3} \cdot \sqrt{x^3} + 2\sqrt{x} + c =$

$= \frac{2}{5} x^2 \sqrt{x} + \frac{2}{3} x \sqrt{x} + 2\sqrt{x} + c$

D) Halle la integral indefinida en las preguntas 1 a 10

① $\frac{1}{2}\int 2(2x+5)^2 \cdot dx = \frac{1}{2}\frac{(2x+5)^3}{3} + c = \frac{(2x+5)^3}{6} + c$

② $\frac{1}{-3}\int -3(-3x+5)^3 \cdot dx = -\frac{1}{3} \cdot \frac{(-3x+5)^4}{4} + c = \frac{-(-3x+5)^4}{12} + c$

③ $2\int \frac{1}{2} e^{\frac{1}{2}x-3} \cdot dx = 2 \cdot e^{\frac{1}{2}x-3} + c$

④ $\frac{1}{5}\int 5 \cdot \frac{1}{5x+4} \cdot dx = \frac{1}{5} \cdot \ln|5x+4| + c$

⑤ $\int \frac{3}{7-2x} \cdot dx = 3 \cdot \frac{1}{-2} \cdot \int -2 \cdot \frac{1}{7-2x} \cdot dx = -\frac{3}{2} \cdot \ln|7-2x| + c$

⑥ $\int 4 \cdot e^{2x+1} dx = 4 \cdot \frac{1}{2} \cdot \int 2 \cdot e^{2x+1} \cdot dx = 2 \cdot e^{2x+1} + c$

⑦ $\int 6(4x-3)^7 \cdot dx = 6 \cdot \frac{1}{4}\int 4 \cdot (4x-3)^7 \cdot dx = \frac{3}{2} \cdot \frac{(4x-3)^8}{8} + c =$

$= \frac{3(4x-3)^8}{16} + c$

⑧ $\frac{1}{7}\int 7 (7x+2)^{1/2} \cdot dx = \frac{1}{7} \frac{(7x+2)^{3/2}}{\frac{3}{2}} + c = \frac{2}{21} \cdot (7x+2)^{3/2} + c$

⑨ $\int \left(e^{4x} + \frac{4}{3x-5}\right) dx = \frac{1}{4}\int 4 \cdot e^{4x} \cdot dx + 4\frac{1}{3}\int \frac{3}{3x-5} \cdot dx =$

$= \frac{1}{4} \cdot e^{4x} + \frac{4}{3} \cdot \ln|3x-5| + c$

⑩ $\frac{1}{4} \cdot \frac{2}{3}\int 4 \cdot (4x-5)^{-3} = \frac{2}{12} \cdot \frac{(4x-5)^{-2}}{-2} + c = \frac{-1}{12 \cdot (4x-5)^2} + c$

E

① $\int (2x^2+5)^2 (4x) \cdot dx = \dfrac{(2x^2+5)^3}{3} + c$; $\int u^n \cdot u' \cdot dx = \dfrac{u^{n+1}}{n+1} + c$

② $\int \dfrac{3x^2+2}{x^3+2x} \cdot dx = \ln |x^3+2x| + c$

③ $\int (6x+5)\sqrt{3x^2+5x}\, dx = \int (6x+5) \cdot (3x^2+5x)^{1/2} \cdot dx = \dfrac{(3x^2+5x)^{3/2}}{\frac{3}{2}} + c =$

$= \dfrac{2}{3} \sqrt{(3x^2+5x)^3} + c$

④ $\int 4x^3 \cdot e^{x^4} \cdot dx = e^{x^4} + c$; $\int e^u \cdot u' \cdot dx = e^u + c$

⑤ $\int \dfrac{2x+3}{(x^2+3x+1)^2} \cdot dx = \int (2x+3) \cdot (x^2+3x+1)^{-2} \cdot dx = \dfrac{(x^2+3x+1)^{-1}}{-1} + c =$

$= \dfrac{-1}{(x^2+3x+1)} + c$

⑥ $\int \dfrac{e^{\sqrt{x}}}{2\sqrt{x}} \cdot dx = \int \dfrac{1}{2\sqrt{x}} \cdot e^{\sqrt{x}} \cdot dx = e^{\sqrt{x}} + c$; $\int e^u \cdot u' \cdot dx = e^u + c$

⑦ $\frac{1}{6}\int 6x^2 (2x^3+5)^4 \cdot dx = \dfrac{(2x^3+5)^5}{5} + c$

⑧ $\int \dfrac{2x+1}{\sqrt[4]{x^2+x}} \cdot dx = \int (2x+1) \cdot (x^2+x)^{-1/4} \cdot dx = \dfrac{(x^2+x)^{3/4}}{\frac{3}{4}} + c =$

$= \dfrac{4}{3} \cdot \sqrt[4]{(x^2+x)^3} + c$

(9) $\int (8x^3-4x)(x^4-x^2)^3 \cdot dx = \int 2(4x^3-2x)(x^4-x^2)^3 \cdot dx =$

$= 2 \cdot \frac{(x^4-x^2)^4}{4} + C = \frac{1}{2}(x^4-x^2)^4 + C$

(10) $\int \frac{4-3x^2}{x^3-4x} \cdot dx = \int \frac{-(3x^2+4)}{x^3-4x} \cdot dx = -\int \frac{(3x^2+4)}{x^3+4x} \cdot dx =$

$= -\ln|x^3+4x| + C$

Integrales en problemas de BI (NM) desde 2014

(1) $\int \sin 3x \cdot \cos 3x \cdot dx = \cancel{\frac{1}{3}\int 3\sin 3x \cdot \cos 3x \cdot dx = \frac{1}{3}}$

$= \int (\sin 3x)^1 \cdot \cos 3x \cdot dx = \frac{1}{6}\int 6 \sin 3x \cdot \cos 3x \cdot dx = \frac{1}{6} \cdot \frac{(\sin 3x)^2}{\cancel{2}} + C =$

$= \frac{(\sin 3x)^2}{6} + C$

$(\sin 3x)^2 \underset{\int}{\overset{d}{\rightleftarrows}} 2\sin 3x \cdot \cos 3x \cdot 3$

(2) $\int x^2 \cdot dx = \frac{x^3}{3} + C$

(3) $\frac{1}{2}\int 2\cos 2x \cdot dx = \text{sen } 2x + C$

$\text{sen } 2x \underset{\int}{\overset{d}{\rightleftarrows}} \cos 2x \cdot 2$

(4) $h(x) = \int h'(x) \cdot dx = \int 4 \cdot \cos 2x \cdot dx = 4 \cdot \frac{1}{2}\int 2\cos 2x \cdot dx = 2 \cdot \text{sen } 2x + C$

(5) $\int \frac{2x}{x^2+5} \cdot dx = \ln|x^2+5| + C$

(6) $\int (5-x^2)\, dx = 5x - \frac{x^3}{3} + C$

7 $\frac{1}{2}\int \frac{2x}{x^2+1}\cdot dx = \frac{1}{2}\ln|x^2+1| + c$

8 $\int (-x^4+2x^3-1)\cdot dx = -\frac{x^5}{5} + 2\frac{x^4}{4} - x + c$

9 a) $g'(x) = \frac{(\ln x)'\cdot x - x'\cdot \ln x}{x^2} = \frac{\frac{1}{\cancel{x}}\cdot \cancel{x} - \ln x}{x^2} = \frac{1-\ln x}{x^2}$

b) $\int \frac{\ln x}{x}\cdot dx = \frac{1}{2}\int 2\ln x\cdot \frac{1}{x}\cdot dx = \frac{1}{2}\cdot(\ln x)^2 + c$

$(\ln x)^2 \xrightarrow{d} 2\cdot \ln x\cdot \frac{1}{x}$

10 a) $\int \frac{9}{x+2}\cdot dx = 9\int \frac{1}{x+2}\cdot dx = 9\cdot \ln|x+2| + c$

b) $\int 3x^2\cdot dx = 3\int x^2\cdot dx = \cancel{3}\frac{x^3}{\cancel{3}} + c = x^3 + c$

11 $f(x) = \int f'(x)\cdot dx = \int (6x^2-5)\,dx = 6\frac{x^3}{3} - 5x + c = 2x^3 - 5x + C$

12 $\int 2\ln(x-3)\,dx =$ POR PARTES: $\int u\cdot dv = u\cdot v - \int v\cdot du$

$= 2\int \ln(x-3)\,dx = 2\left((\ln(x-3))\cdot x - \int x\cdot \frac{1}{x-3}\cdot dx\right) =$

$u = \ln(x-3)$ | $du = \frac{1}{x-3}\cdot dx$

$dv = dx$ | $v = x$

$2\cdot\left(x\cdot \ln|x-3| - (x + 3\ln|x-3|)\right) =$

$= 2\cdot(x\ln|x-3| - x - 3\ln|x-3|) + C$

A $\int \frac{x}{x-3}\cdot dx = \int \left(1 + \frac{3}{x-3}\right)dx = x + 3\cdot \ln|x-3|$

$$\begin{array}{r|l} x & x-3 \\ \hline -x+3 & 1 \\ 0+3 & \end{array}$$

13 $\int x^2 \cdot dx = \frac{x^3}{3} + c$

$\int 3\ln(x+1)\cdot dx = 3\int \ln(x+1)\cdot dx =$ POR PARTES:

$u = \ln(x+1)$ | $du = \frac{1}{x+1}$

$dv = dx$ | $v = x$

$= 3\cdot\left(x\cdot \ln(x+1) - \underbrace{\int x\cdot\frac{1}{x+1}\cdot dx}_{A}\right) = 3\cdot\left(x\cdot\ln(x+1) - x + \ln|x+1| + c\right)$

A $\int \frac{x}{x+1}\cdot dx = \int\left(1 + \frac{-1}{x+1}\right)dx = x - \ln|x+1|$

$$\begin{array}{r|l} x & x+1 \\ \hline -x-1 & 1 \\ 0\ \ -1 & \end{array}$$

14 a/ $\int x\cdot \underbrace{e^{-x}}_{f(x)}\cdot dx = x\cdot(-e^{-x}) - \int -e^{-x}\cdot dx =$ POR PARTES:

$u = x$ | $du = dx$

$dv = e^{-x}\cdot dx$ | $v = -\int -e^{-x}\cdot dx = -e^{-x}$

$e^{-x} \underset{i}{\overset{d}{\rightleftarrows}} e^{-x}(-1)$

$= \boxed{-e^{-x}\cdot x - e^{-x} + c}$

$= e^{-x}(-x-1) + c$

b/ $\int(-3f(x) + 1)\,dx = -3\int f(x)\cdot dx + \int dx =$

$= -3\cdot\left(e^{-x}(-x-1) \cancel{+c}\right) + x + C$

15 a) $\int x \cdot e^{x^2-1} \cdot dx =$ $e^{x^2-1} \xrightarrow{d} e^{x^2-1} \cdot 2x$, $\xleftarrow{i}$

$= \frac{1}{2}\int 2 \cdot x \cdot e^{x^2-1} \cdot dx = \frac{1}{2} e^{x^2-1} + c$

b) $f(x) = \int f'(x) \cdot dx = \frac{1}{2} e^{x^2-1} + c$

Si $f(-1) = 3 \Rightarrow \frac{1}{2} \cdot e^{(-1)^2-1} + c = 3$; $\frac{1}{2} \cdot e^0 + c = 3$; $c = \frac{5}{2}$

luego $\boxed{f(x) = \frac{1}{2} e^{x^2-1} + \frac{5}{2}}$

16 $f(x) = \int \frac{3x^2}{(x^3+1)^5} \cdot dx = \int (x^3+1)^{-5} \cdot 3x^2 \cdot dx = \frac{(x^3+1)^{-4}}{-4} + c =$

$= \frac{-1}{(x^3+1)^4} + c$

Si $f(0) = 1 \Rightarrow \frac{-1}{(0^3+1)^4} + c = 1 \Rightarrow c = 2$

luego $\boxed{f(x) = \frac{-1}{(x^3+1)^4} + 2}$

17 a) $\int \ln x \cdot dx = (\ln x) \cdot x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \cdot dx = \boxed{x \cdot \ln x - x + c}$ POR PARTES

$u = \ln x \quad | \quad du = \frac{1}{x}\, dx$

$dv = dx \quad | \quad v = x$

17 b)

$$\int\left(3+\ln\left(\frac{x}{2}\right)\right)\cdot dx = \int 3\,dx + \int \ln\frac{x}{2}\cdot dx = 3x + \int \ln\frac{x}{2}\cdot dx =$$

A

A $\int \ln\frac{x}{2}\cdot dx = \ln\left(\frac{x}{2}\right)\cdot x - \int \cancel{x}\cdot\frac{2}{\cancel{x}}\cdot dx = \boxed{x\ln\left(\frac{x}{2}\right) - 2x}$

Por partes

$u = \ln\frac{x}{2}$ | $u = \frac{1}{\frac{x}{2}}\cdot dx = \frac{2}{x}\cdot dx$

$dv = dx$ | $v = x$

$= 3x + x\cdot\ln\left(\frac{x}{2}\right) - 2x + C$

18)

$$\int(-0'5x^4 + 3x^2 + 2x)\,dx = -0'5\cdot\frac{x^5}{5} + 3\frac{x^3}{3} + 2\frac{x^2}{2} + C =$$

$$= -\frac{1}{10}x^5 + x^3 + x^2 + C$$

Integrales en Exámenes EBAU y PAU Asturias

① $F(x) = \int f(x)\cdot dx = \int(x^3 - 3x^2 + 2x)\,dx = \frac{x^4}{4} - \cancel{3}\frac{x^3}{\cancel{3}} + \cancel{2}\frac{x^2}{\cancel{2}} + C$

Si $F(2) = 1 \Rightarrow \frac{2^4}{4} - 2^3 + 2^2 + C = 1$; $4 - 8 + 4 + C = 1$; $\boxed{C = 1}$

luego $F(x) = \boxed{\frac{x^4}{4} - x^3 + x^2 + 1}$

② $F(x) = \int f(x)\cdot dx = \int(5 + 6x + 24x^2)\,dx = 5x + 6\frac{x^2}{2} + 24\frac{x^3}{3} + C$

Si $F(2) = 90 \Rightarrow 5\cdot 2 + 3\cdot 2^2 + 8\cdot 2^3 + C = 90 \Rightarrow \boxed{C = 4}$

luego $F(x) = \boxed{5x + 3x^2 + 8x^3 + 4}$

3 $F(x)=\int(4x-x^3)\,dx = \frac{4x^2}{2}-\frac{x^4}{4}+C$

Si $F(2)=7 \Rightarrow 2\cdot 2^2-\frac{2^4}{4}+C=7 \Rightarrow \boxed{C=3}$

luego $F(x) = \boxed{2x^2-\frac{x^4}{4}+3}$

4 $F(x)=\int(x^3-5x^2+4x)\,dx = \frac{x^4}{4}-5\frac{x^3}{3}+4\frac{x^2}{2}+C$

....

5 $F(x)=\int(0'6-0'01x)\,dx = 0'6x-0'01\frac{x^2}{2}+C$

...

6 $F(x)=\int\frac{20}{(x+2)^2}\cdot dx = 20\cdot\int(x+2)^{-2}\cdot dx = 20\cdot\frac{(x+2)^{-1}}{-1}+C =$

$= \frac{-20}{x+2}+C$ ....

7 $F(x)=\int\frac{10}{(x+1)^2}\cdot dx = 10\int(x+1)^{-2}\cdot dx = 10\frac{(x+1)^{-1}}{-1}+C =$

$= \frac{-10}{x+1}+C$ ....

8 $F(x)=\int(3+8x+15x^2)\,dx = 3x+8\frac{x^2}{2}+15\frac{x^3}{3}+C$

...

MÉTODO DE INTEGRACIÓN POR PARTES O INTEGRAL DE UN PRODUCTO

$$(u \cdot v)' = \begin{cases} u \cdot dv + v \cdot du \\ \text{es decir } u \cdot v' + v \cdot u' \end{cases}$$

prácticamente.

integrando:

$$\int (u \cdot v)' = \int u \cdot dv + \int v \cdot du$$

$$u \cdot v = \int u \cdot dv + \int v \cdot du$$

Despejando:

$$\boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$$

Ejemplo problema 24 pag. 224:

$\int x \cdot Lx \cdot dx$ (producto) $\rightarrow$ llamamos $\begin{cases} Lx = u \\ x \cdot dx = dv \end{cases}$

Si no saliera probamos de otro modo.

derivando e integrando respectivamente queda:

$$\begin{cases} d\,Lx = du \rightarrow \dfrac{1}{x} \cdot dx = du \\ \int x \cdot dx = \int dv \rightarrow \dfrac{x^2}{2} = v \end{cases}$$

Sustituyendo en la fórmula queda:

$$\int \underbrace{x \cdot \underset{u}{Lx} \cdot dx}_{dv} = Lx \cdot \frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{x}} \cdot dx = \boxed{Lx \cdot \frac{x^2}{2} - \frac{1}{4} x^2} + C$$

Ejemplo. ejercicio 25. pag. 224

$$\int \underset{u}{x} \cdot \underbrace{e^{4x} \cdot dx}_{dv}$$

$u = x \longrightarrow du = dx$

$dv = e^{4x} \cdot dx \longrightarrow \int dv = v = \int e^{4x} \cdot dx = \frac{1}{4} e^{4x}$

$$= x \cdot \frac{1}{4} e^{4x} - \int \frac{1}{4} e^{4x} \cdot dx = \boxed{\frac{1}{4} x \cdot e^{4x} - \frac{1}{16} e^{4x}} + C$$

Ejercicios:

1. $\int x \cdot Lx \cdot dx =$ Hecha en el ejemplo
2. $\int x \cdot e^{4x} \cdot dx =$ Hecho en el ejemplo
3. $\int x^2 e^x \cdot dx =$
4. $\int x^2 \cdot e^{-x} \cdot dx$
5. $\int x^2 \cdot e^{2x} \cdot dx$
6. $\int x^2 \cdot \cos x \cdot dx$
7. $\int x^2 \cdot \operatorname{sen} x \cdot dx$
8. $\int e^x \cdot \operatorname{sen} x \cdot dx$
9. $\int e^{-x} \cdot \cos x \cdot dx$
10. $\int (x-1) \cdot e^x \cdot dx$

Integrales por partes

③ $\int x^2 \cdot e^x \cdot dx = x^2 \cdot e^x - \int e^x \cdot 2x \cdot dx = x^2 \cdot e^x - 2 \int x \cdot e^x \cdot dx =$ (A)

$u = x^2$ | $du = 2x\, dx$

$dv = e^x \cdot dx$ | $v = e^x$

(A) $u = x$ | $du = dx$

$dv = e^x \cdot dx$ | $v = e^x$

$= x^2 \cdot e^x - 2\left(x \cdot e^x - \int e^x \cdot dx\right) =$

$\boxed{= x^2 e^x - 2xe^x + 2e^x + C}$

✓

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④ $\int x^2 \cdot e^{-x} \cdot dx = x^2 \cdot (-e^{-x}) - \int -e^{-x} \cdot 2x \cdot dx = -x^2 e^{-x} + 2 \int x \cdot e^{-x} \cdot dx =$ (A)

$u = x^2$ | $du = 2x \cdot dx$

$dv = e^{-x} \cdot dx$ | $v = -\int -e^{-x} \cdot dx = -e^{-x}$

$e^{-x} \xrightarrow{d} e^{-x}(-1)$

$\xleftarrow{i}$

(A) $u = x$ | $du = dx$

$dv = e^{-x} dx$ | $v = -e^{-x}$

$= -x^2 \cdot e^{-x} + 2 \cdot \left(x \cdot (-e^{-x}) - \int -e^{-x} \cdot dx\right) =$

$\boxed{= -x^2 \cdot e^{-x} - 2x \cdot e^{-x} - 2e^{-x} + C} =$

$\boxed{= e^{-x}(-x^2 - 2x - 2) + C}$

---

⑤ $\int x^2 \cdot e^{2x} \cdot dx = x^2 \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{\cancel{2}} \cancel{2}x \cdot dx = x^2 \cdot \frac{e^{2x}}{2} - \int x \cdot e^{2x} \cdot dx =$ (A)

$u = x^2$ | $du = 2x \cdot dx$

$dv = e^{2x} dx$ | $v = \frac{1}{2} \int 2 e^{2x} dx = \frac{e^{2x}}{2}$

(A) $u = x$ | $du = dx$

$dv = e^{2x} \cdot dx$ | $v = \frac{e^{2x}}{2}$

$= \frac{1}{2} x^2 \cdot e^{2x} - \left(x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot dx\right) =$

$= \frac{1}{2} x^2 \cdot e^{2x} - \frac{1}{2} x \cdot e^{2x} + \frac{1}{2} \int e^{2x} dx =$

$\frac{e^{2x}}{2}$

$\boxed{= \frac{1}{2} e^{2x} \cdot \left(x^2 - x + \frac{1}{2}\right) + C}$

$= \frac{1}{2} x^2 \cdot e^{2x} - \frac{1}{2} x \cdot e^{2x} + \frac{1}{4} e^{2x} + C$

(14)

(6) $\int x^2 \cdot \cos x \cdot dx = x^2 \cdot \text{sen}\, x - \int \text{sen}\, x \cdot 2x \cdot dx = x^2 \cdot \text{sen}\, x - 2\int x \cdot \text{sen}\, x \cdot dx =$ (A)

$u = x^2 \quad | \quad du = 2x \cdot dx$

$dv = \cos x \cdot dx \quad | \quad v = \text{sen}\, x$

(A) $u = x \quad | \quad du = dx$

$dv = \text{sen}\, x \quad | \quad v = -\int -\text{sen}\, x \cdot dx = -\cos x$

menos

$= x^2 \cdot \text{sen}\, x - 2\left(x \cdot (-\cos x) \cancel{+} \int -\cos x \cdot dx\right) =$

$\boxed{= x^2 \cdot \text{sen}\, x + 2x \cos x \cancel{+} 2\,\text{sen}\, x + C}$

menos

$\int -\cos x \cdot dx = -\text{sen}\, x$

✓

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(7) $\int x^2 \cdot \text{sen}\, x \cdot dx = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x \cdot dx = -x^2 \cdot \cos x + 2\int x \cdot \cos x \cdot dx =$ (A)

$u = x^2 \quad | \quad du = 2x \cdot dx$

$dv = \text{sen}\, x \cdot dx \quad | \quad v = \int \text{sen}\, x \cdot dx = -\cos x$

(A) $x = u \quad | \quad du = dx$

$dv = \cos x \cdot dx \quad | \quad v = \int \cos x \cdot dx = \text{sen}\, x$

$= -x^2 \cdot \cos x + 2\left(x \cdot \text{sen}\, x - \int \text{sen}\, x \cdot dx\right) =$

$\boxed{= -x^2 \cdot \cos x + 2x \cdot \text{sen}\, x + 2\cos x + C}$

✓

---

(8) $\int e^x \cdot \text{sen}\, x \cdot dx = \text{sen}\, x \cdot e^x - \int e^x \cdot \cos x \cdot dx = \text{sen}\, x \cdot e^x - \left(\cos x \cdot e^x - \int e^x \cdot (-\text{sen}\, x) \cdot dx\right) =$ (A)

$u = \text{sen}\, x \quad | \quad du = \cos x \cdot dx$

$dv = e^x \cdot dx \quad | \quad v = e^x$

(A) $u = \cos x \quad | \quad du = -\text{sen}\, x \cdot dx$

$dv = e^x \cdot dx \quad | \quad v = e^x$

$= \text{sen}\, x \cdot e^x - \cos x \cdot e^x - \int e^x \cdot \text{sen}\, x \cdot dx$

Se pasa para el lado izquierdo sumando y:

$2\int e^x \cdot \text{sen}\, x = e^x \cdot \text{sen}\, x - e^x \cdot \cos x$

$$\boxed{\int e^x \cdot \text{sen}\, x = \frac{e^x(\text{sen}\, x - \cos x)}{2} + C}$$

✓

⑨ $\int e^{-x}\cdot \cos x\cdot dx = \cos x\cdot(-e^{-x}) - \int -e^{-x}\cdot(-\operatorname{sen} x)\cdot dx = -e^{-x}\cdot\cos x \cancel{+} \int e^{-x}\cdot \operatorname{sen} x\cdot dx =$ ⑮

menos Ⓐ

$u=\cos x \quad | \quad du=-\operatorname{sen} x\cdot dx$

$dv=e^{-x}\cdot dx \quad | \quad v=\int e^{-x}\cdot dx = -e^{-x}$

Ⓐ $u=\operatorname{sen} x \quad | \quad du=\cos x\cdot dx$

$dv=e^{-x}\cdot dx \quad | \quad v=\int e^{-x}\cdot dx=-e^{-x}$

$= -e^{-x}\cdot\cos x - \left(\operatorname{sen} x\cdot(-e^{-x}) - \int -e^{-x}\cdot\cos x\cdot dx\right) =$

$= -e^{-x}\cdot\cos x + \operatorname{sen} x\cdot e^{-x} - \int e^{-x}\cdot\cos x\cdot dx$

Se pasa sumando al lado de la izquierda

$2\int e^{-x}\cdot\cos x\cdot dx = e^{-x}\cdot(\operatorname{sen} x - \cos x)$

$$\boxed{\int e^{-x}\cdot\cos x\cdot dx = \frac{e^{-x}(\operatorname{sen} x-\cos x)}{2} + C}$$

✓

⑩ $\int (x-1)\cdot e^{x}\cdot dx = (x-1)\cdot e^{x} - \int e^{x}\cdot dx = (x-1)\cdot e^{x} - e^{x} + C =$

$u=x-1 \quad | \quad du=dx$

$dv=e^{x}\cdot dx \quad | \quad v=e^{x}$

$= e^{x}\left((x-1)-1\right)+C =$

$= \boxed{e^{x}(x-2)+C}$