Límites y derivadas

(1) a) $f'(x) = \dfrac{-1}{x^2}$

$f''(x) = \dfrac{-2x\cdot(-1)}{x^4} = \dfrac{2x}{x^4} = \dfrac{2}{x^3}$

$f'''(x) = \dfrac{-3x^2\cdot 2}{x^6} = \dfrac{-6x^2}{x^6} = \dfrac{-6}{x^4}$

$f^{IV}(x) = \dfrac{-4x^3\cdot(-6)}{x^8} = \dfrac{+24x^3}{x^8} = \dfrac{24}{x^5}$

$\dfrac{2}{x^3}$

$-\dfrac{2\cdot 3}{x^4}$

$\dfrac{2\cdot 3\cdot 4}{x^5}$

$\vdots$

b) $f^{(n)}(x) = \boxed{\dfrac{(-1)^n\cdot n!}{x^{n+1}}}$

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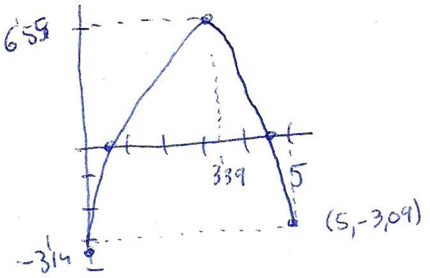
(2) a) $y=0 \longrightarrow 0 = 4x - e^{x-2} - 3 \longrightarrow$ con C.G. $x_1 = 0'827$ $x_2 = 4'78$

b)



c) $f'(3)$ = gradiente en $x=3 \Rightarrow$ con C.G. $\rightarrow \boxed{f'(3) = 1'28}$

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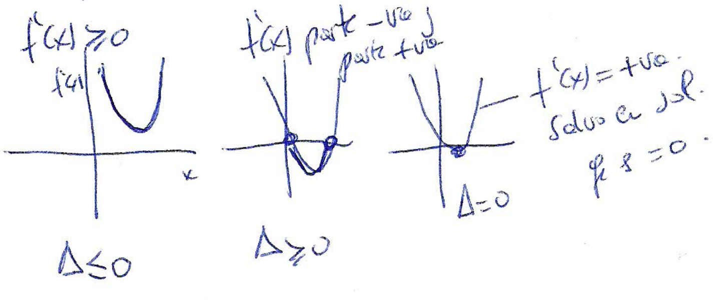
(3) a) $f'(x) = 3px^2 + 2px + q$

b) Si $f'(x) \geq 0 \rightarrow$ es una f. cuadrática $\rightarrow \Delta \leq 0 \rightarrow$ No se trata de los fórmulas sino de la función. (ver dibujo abajo).

$\Delta = 4p^2 - 4\cdot 3p\cdot q = 4p^2 - 12pq \Rightarrow 4p(p-3q) \leq 0$ dividiendo entre 4

$p(p-3q) \leq 0$; $p^2 - 3pq \leq 0 \rightarrow \boxed{p^2 \leq 3pq}$ c.q.d.



4) la pendiente de la tangente a $x=2$ en $x=2$

$f'(2)$; $f'(x)=\dfrac{g'(x)\cdot h(x)-h'(x)\cdot g(x)}{[h(x)]^2}=\dfrac{5\cdot 6-2\cdot 18}{6^2}=-\dfrac{1}{6}$

pendiente de la Normal (perpendicular) a la tg. $\rightarrow \dfrac{6}{1}=\boxed{6}$

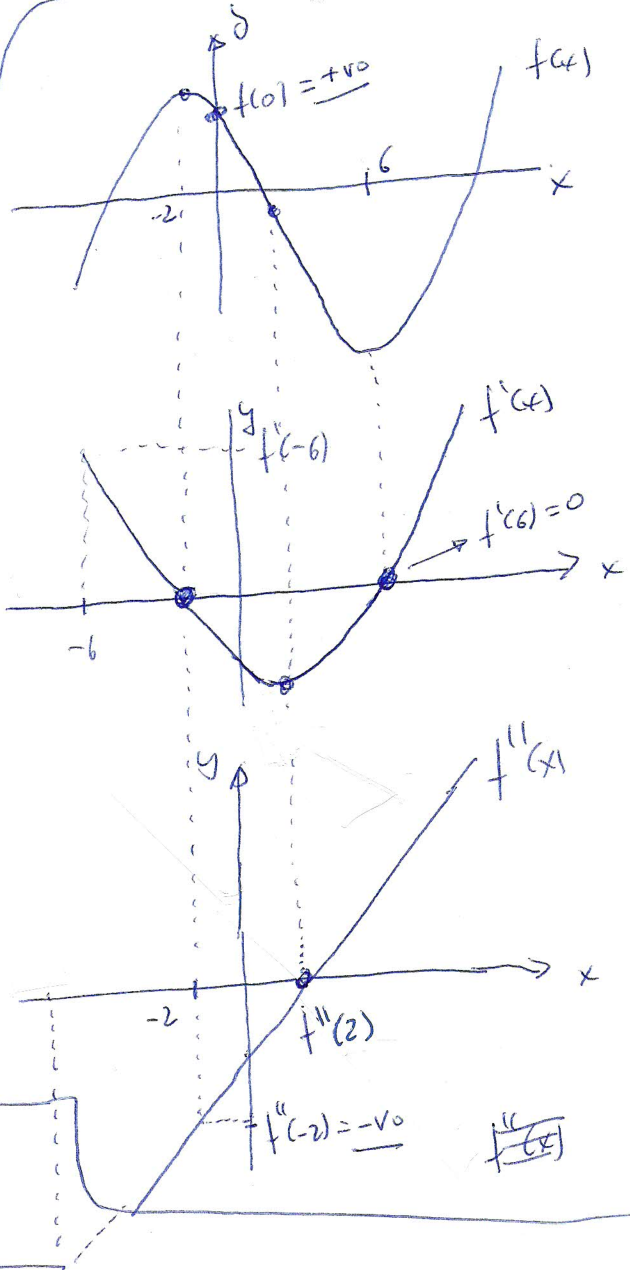
Ec. recta: $y-y_p=m(x-x_p)$ $\boxed{P(2,3)}$

$\boxed{y-3=6(x-2)}$ $f(2)=\dfrac{g(2)}{h(2)}=\dfrac{18}{6}=\boxed{3}$

$y=6x-12+3$

$\boxed{y=6x-9}$

5) a)



b)

$f(0)=?=+vo$

$f'(6)=0$

$f''(-2)=-vo$

$f''(-2)<f'(6)<f(0)$

6) a) $f'(x)=\dfrac{2(x^2+5)-2x(2x)}{(x^2+5)^2}=$

$=\dfrac{2x^2+10-4x^2}{(x^2+5)^2}=\boxed{\dfrac{10-2x^2}{(x^2+5)^2}}$ c.q.d.