① Los nºs racionales son los se pueden expresar como cociente de dos números enteros. Incluirán entonces a los enteros, a los decimales limitados y a los decimales periódicos. Estarán excluidos los de infinitas cifras no periódicas.

a) Verdadero → $\text{nº entero} = \dfrac{\text{nº entero}}{1}$ ✓

b) Falso. Hay nº reales que no son racionales, pero no al revés. Todos los cocientes de nº enteros se pueden situar sobre la recta real utilizando el teorema de Tales.

c) Falso. Es un cociente, pero no de dos números enteros.

d) Falso. Si un nº es racional, podrá expresarse en forma de cociente de dos enteros, cuya división o es finita o tiene periodo, nunca las infinitas cifras decimales no periódicas propias de los irracionales.

②

Racionales no enteros: $\boxed{2'\overline{83} \; ; \; -\frac{3}{4} \; ; \; 3'25}$

Enteros no naturales: $\boxed{\sqrt[3]{-8}+1=-1 \; ; \; -8/4=-2}$

Irracionales: $\boxed{\sqrt{5} \; ; \; \frac{\pi}{4} \; ; \; 1'020020002\ldots}$

Naturales: $\boxed{\sqrt[4]{81}=3 \; ; \; (\sqrt{5}+1)(\sqrt{5}-1)=4}$

③ a) FALSO. Por ejemplo $\pi + (-\pi) = 0$

b) FALSO. Por ejemplo $(\sqrt{5}+1)(\sqrt{5}-1) = 4$

④

$$\begin{array}{l|l} 13 & 6 \\ \hline 10 & 2'16\ldots \\ \;\,40 & \\ \;\;\;4 & \end{array}$$

$$\frac{13}{6} = 2'1\overline{6} \simeq 2'17$$

$$ER = \frac{\left|\frac{13}{6} - 2'17\right|}{13/6} = \frac{\left|\frac{13-13'02}{6}\right|}{13/6} = \frac{\frac{0'02}{6}}{\frac{13}{6}} = \frac{0'02}{13} = \frac{2}{13}\,\% = \boxed{0'15\%}$$

⑤ a) Primer Telémetro:

$$EA = |14'48 - 14'39| = \boxed{0'09\ m}$$

$$ER = \frac{0'09}{14'39} \cdot 100 = \boxed{0'625\%}$$

Segundo Telémetro:

$$EA = |7'85 - 7'92| = \boxed{0'07\ m}$$

$$ER = \frac{0'07}{7'92} \cdot 100 = \boxed{0'884\%}$$

b) Es más preciso el primer telémetro porque el error relativo es inferior, aunque sucede lo contrario con el error absoluto.

⑥

$$EA \le \boxed{0'0005\ mm}$$

$$ER \le \frac{0'0005}{2'318 - 0'0005} \cdot 100 = \boxed{0'02\%}$$

⑦

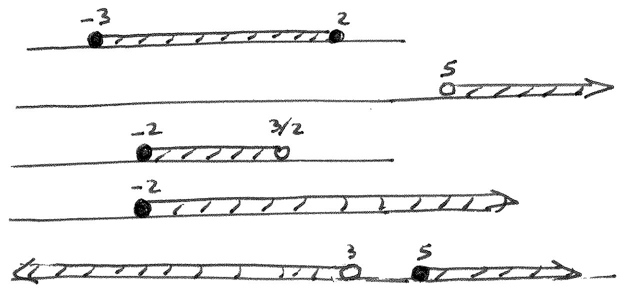
$[-3,2]$

$(5,+\infty)$

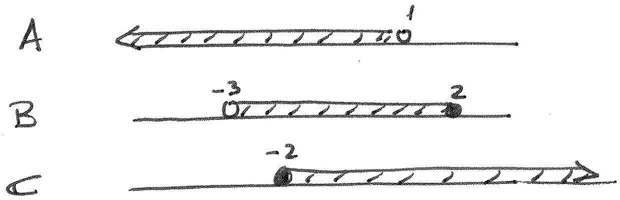
$[-2,\, 3/2)$

$[-2,+\infty)$

$(-\infty,3) \cup [5,+\infty)$



(8)



$A \cap B = (-3, 1)$

$A \cup B = (-\infty, 2]$

$A \cap C = [-2, 1)$

$A \cup C = \mathbb{R}$

$A \cap B \cap C = [-2, 1)$

(9) $\left(\frac{3}{5} - 2\right)^{-1} : \frac{5^2}{6} + \frac{1}{7} = \boxed{-\frac{1}{35}}$

$\sqrt[3]{7\sqrt{3} + 8^4} = \boxed{1'602 \cdot 10^1}$

$\frac{\log\left(3'5 - \frac{2}{3}\right)}{\ln\left(1'2^3\right)} = \boxed{0'827}$

$(4'60 \cdot 10^6) + (5'34 \cdot 10^4) : (1'02 \cdot 10^{-2}) = \boxed{9'84 \cdot 10^6}$

(10) Masa del sol $= 1'670 \cdot 10^{-24} \cdot 1'191 \cdot 10^{57} = 1'989 \cdot 10^{33}$ g $= \boxed{1'989 \cdot 10^{30} \text{ Kg}}$

(11) $1'2 \cdot 10^{11}$ segundos $= 1'2 \cdot 10^{11} \cancel{\text{seg}} \cdot \frac{1 \text{ min}}{60 \cancel{\text{seg}}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ día}}{24 \text{ h}} \cdot \frac{1 \text{ año}}{365 \text{ días}} = \boxed{3'805 \cdot 10^3 \text{ años}}$

(12) Año-luz $= 299.792.458 \ \frac{m}{sg} \cdot 1$ año $= 299.792.458 \ \frac{m}{\cancel{sg}} \cdot 1 \cancel{\text{año}} \cdot \frac{365 \text{ días}}{1 \text{ año}} \cdot \frac{24 \text{ h}}{1 \text{ día}} \cdot \frac{60 \text{ min}}{1 \text{ h}} \cdot \frac{60 \text{ sg}}{1 \text{ min}}$

$= 9'45 \cdot 10^{15}$ m $= \boxed{9'45 \cdot 10^{12} \text{ Km}}$

(13) $5'21\overline{3} - 3'8\overline{7} = \frac{5213 - 521}{900} - \frac{387 - 38}{90} = \frac{4692}{900} - \frac{349}{90} = \boxed{\frac{601}{450}} = \boxed{1'33\overline{5}}$

(14)

$81 = 3^4$

$0'5 = 2^{-1}$

$\frac{1}{25} = 5^{-2}$

$\sqrt[3]{9} = 3^{2/3}$

$\frac{1}{\sqrt{8}} = 2^{-3/2}$

(15) $\sqrt{5} + \sqrt{45} + \sqrt{180} - \sqrt{80} = \sqrt{5} + 3\sqrt{5} + 6\sqrt{5} - 4\sqrt{5} = \boxed{6\sqrt{5}} = \boxed{\sqrt{180}}$

$\sqrt[3]{-54} + 2\sqrt[3]{16} = -3\sqrt[3]{2} + 4\sqrt[3]{2} = \boxed{\sqrt[3]{2}}$

(16) $\sqrt[6]{27} = \sqrt[6]{3^3} = \boxed{\sqrt{3}}$

$\sqrt[6]{125} = \sqrt[6]{5^3} = \boxed{\sqrt{5}}$

$\frac{3\sqrt{512} + 5\sqrt{32}}{\sqrt{50} - \sqrt{18}} = \frac{48\sqrt{2} + 20\sqrt{2}}{5\sqrt{2} - 3\sqrt{2}} = \frac{68\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}} = \boxed{34}$

(17)

$$\sqrt{5} = \boxed{\sqrt[30]{5^{15}}}$$

$$\sqrt[5]{2^3} = \boxed{\sqrt[30]{2^{18}}}$$

$$\sqrt[15]{7^2} = \boxed{\sqrt[30]{7^4}}$$

(18) La raíz cuadrada de una suma no es igual a la suma de raíces cuadradas.

Por eso: $\sqrt{4'25} \neq \sqrt{4} + \sqrt{0'25} \Rightarrow \sqrt{4'25} \neq 2'5$

(19)

$$\frac{\sqrt{27} \cdot \sqrt{\sqrt[5]{81}}}{3^5 \cdot (\sqrt[3]{3})^4 \cdot 9^{-2}} = \frac{3^{3/2} \cdot 3^{4/10}}{3^5 \cdot 3^{4/3} \cdot 3^{-4}} = 3^{\frac{3}{2}+\frac{4}{10}-5-\frac{4}{3}+4} = \boxed{3^{-13/30}} = \boxed{\frac{1}{\sqrt[30]{3^{13}}}}$$

$$\frac{2^{3/2} \cdot 8^0 \cdot 4^{-1/3}}{2^{-1} \cdot \sqrt[3]{2}} = \frac{2^{3/2} \cdot 1 \cdot 2^{-2/3}}{2^{-1} \cdot 2^{1/3}} = 2^{\frac{3}{2}-\frac{2}{3}+1-\frac{1}{3}} = \boxed{2^{3/2}} = \boxed{\sqrt{2^3}}$$

(20)

$$\frac{3}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \boxed{\frac{\sqrt{6}}{2}}$$

$$\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}\sqrt{3}}{3} = \boxed{\frac{\sqrt{6}}{3}}$$

$$\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} = \boxed{\frac{\sqrt{10}}{2}}$$

$$\frac{6}{\sqrt[3]{2}} = \frac{6\sqrt[3]{2^2}}{\sqrt[3]{2^3}} = \frac{6\sqrt[3]{4}}{2} = \boxed{3\sqrt[3]{4}}$$

$$\frac{2}{1-\sqrt{3}} = \frac{2(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{2(1+\sqrt{3})}{1-3} = \frac{2(1+\sqrt{3})}{-2} = \boxed{-(1+\sqrt{3})}$$

$$\frac{9}{\sqrt{7}-\sqrt{3}} = \frac{9(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{9(\sqrt{7}+\sqrt{3})}{7-3} = \boxed{\frac{9\sqrt{7}+9\sqrt{3}}{4}}$$

(21)

$$\sqrt{\frac{5}{12}} - \sqrt{\frac{10}{6}} = \frac{\sqrt{5}}{2\sqrt{3}} - \frac{\sqrt{10}}{\sqrt{6}} = \frac{\sqrt{5}\sqrt{3}}{6} - \frac{\sqrt{60}}{6} = \frac{\sqrt{15}-\sqrt{60}}{6} = \frac{\sqrt{15}(1-2)}{6} = \boxed{-\frac{\sqrt{15}}{6}}$$

(22)

$$\sqrt[4]{9} + \sqrt{\frac{1}{3}} - \sqrt{\frac{4}{27}} = \sqrt{3} + \frac{1}{\sqrt{3}} - \frac{2}{3\sqrt{3}} = \sqrt{3} + \frac{\sqrt{3}}{3} - \frac{2\sqrt{3}}{9} = \frac{9\sqrt{3}+3\sqrt{3}-2\sqrt{3}}{9} = \boxed{\frac{10\sqrt{3}}{9}} \quad \boxed{\begin{matrix} a=10 \\ b=9 \end{matrix}}$$

(23) $B = 10 \log\left(\frac{I}{10^{-12}}\right)$

a) $I = 10^{-9}\ \frac{w}{m^2} \Rightarrow B = 10 \log\left(\frac{10^{-9}}{10^{-12}}\right) = 10 \log 10^3 = \boxed{30\ \text{db}}$

b) $B = 110$ db $\Rightarrow 110 = 10 \log\left(\frac{I}{10^{-12}}\right)$; $11 = \log\left(\frac{I}{10^{-12}}\right)$; $10^{11} = \frac{I}{10^{-12}}$; $\boxed{I = 10^{-1}\ \frac{w}{m^2}}$

(24) $M = \frac{2}{3} \log\left(\frac{E}{2'5 \cdot 10^4}\right)$

a) $E = 3 \cdot 10^6$ jul $\Rightarrow M = \frac{2}{3} \log\left(\frac{3 \cdot 10^6}{2'5 \cdot 10^4}\right) = \boxed{1'39\ \text{°Richter}}$

b) $M = 8'25$ °Richter $\Rightarrow 8'25 = \frac{2}{3} \log\left(\frac{E}{2'5 \cdot 10^4}\right)$; $12'375 = \log\left(\frac{E}{2'5 \cdot 10^4}\right)$; $10^{12'375} = \frac{E}{2'5 \cdot 10^4}$;

$E = 2'5 \cdot 10^{16'375} = \boxed{5'93 \cdot 10^{16}\ \text{julios}}$

(25) $x = \log_2 \frac{1}{64} = \log_2 2^{-6} = \boxed{-6}$

$4 = \log_x 81 \rightarrow x^4 = 81 \;;\; x = \sqrt[4]{81} = \boxed{3}$

$-2 = \log_5 x \rightarrow x = 5^{-2} = \boxed{\frac{1}{25}}$

$\log_a 4 + 2\log_a 3 - \frac{1}{2}\log_a 9 = \log_a x \rightarrow \cancel{\log_a} \frac{4\cdot 3^2}{\sqrt{9}} = \cancel{\log_a} x \;;\; x = \frac{4\cdot 3^2}{\sqrt{9}} = \boxed{12}$

(26) $3^x = 6561 \rightarrow \boxed{x=8}$

$2^x = 0'25 \rightarrow \boxed{x=-2}$

$2^{3x+2} = 1024 \rightarrow 3x+2 = 10 \;;\; 3x = 8 \;;\; \boxed{x = \frac{8}{3}}$

$\left(\frac{1}{3}\right)^x = 81 \rightarrow 3^{-x} = 81 \;;\; \boxed{x=-4}$

(27) $3^x = 6000 \rightarrow x = \frac{\log 6000}{\log 3} = \boxed{7'919}$

$2^x = 0'3 \rightarrow x = \frac{\log 0'3}{\log 2} = \boxed{-1'737}$

$2^{3x+2} = 1000 \rightarrow 3x+2 = \frac{\log 1000}{\log 2} \;;\; 3x+2 = 9'966 \;;\; x = \frac{7'966}{3} = \boxed{2'655}$

$\left(\frac{1}{3}\right)^x = 100 \rightarrow x = \frac{\log 100}{\log(1/3)} = \boxed{-4'192}$

(28) a) $5^{2x} - 30\cdot 5^x + 125 = 0$

$t = 5^x \rightarrow t^2 - 30t + 125 = 0 \Rightarrow t = \begin{cases} 5 \Rightarrow 5^x = 5, \; \boxed{x=1} \\ 25 \Rightarrow 5^x = 25, \; \boxed{x=2} \end{cases}$

b) $9^x - 2\cdot 3^{x+2} + 81 = 0 \;;\; 3^{2x} - 2\cdot 3^x \cdot 3^2 + 81 = 0$

$t = 3^x \rightarrow t^2 - 18t + 81 = 0 \rightarrow t = 9 \Rightarrow 3^x = 9, \; \boxed{x=2}$

c) $2^{x-1} + 2^x + 2^{x+1} = 7$

$\frac{2^x}{2} + 2^x + 2^x\cdot 2 = 7$

$t = 2^x \rightarrow \frac{t}{2} + t + 2t = 7 \;;\; t + 2t + 4t = 14, \; 7t = 14, \; t = 2 \Rightarrow 2^x = 2, \; \boxed{x=1}$

(29) $\log_x 9 = 2 \rightarrow x^2 = 9, \; x = \sqrt{9} = \boxed{3}$

$\log_2 x = 3 \rightarrow x = 2^3 = \boxed{8}$

$\log x = -1 \rightarrow x = 10^{-1} = \boxed{0'1}$

$\log_8 2 = x \rightarrow 8^x = 2, \; 2^{3x} = 2 \rightarrow 3x = 1, \; \boxed{x = \frac{1}{3}}$

(30) $\log x + \log 50 = \log 1000$, $50x = 1000$, $\boxed{x = 20}$

$\log x - \log(22-x) = 1$, $\frac{x}{22-x} = 10$, $x = 220 - 10x$; $11x = 220$, $\boxed{x = 20}$

$\log x^3 = \log 6 + 2\log x$, $x^3 = 6x^2$; $x^2(x-6) = 0$, $x = \begin{cases} \cancel{0} \\ \boxed{6} \end{cases}$ No es solución porque no existe logo

(31) $10^{\log_{10} x} = \boxed{x}$

$\log_{10} 10^x = \boxed{x}$

$e^{\ln x} = \boxed{x}$

$\ln e^x = \boxed{x}$

$2^{3\log_2 x} = 2^{\log_2(x^3)} = \boxed{x^3}$