① $x = 12'\overline{90}$

$$100x = 1290'9090\ldots$$
$$x = 12'9090\ldots$$
$$99x = 1278 \qquad x = \frac{1278}{99} = \boxed{\frac{142}{11}}$$

$x = 76'1\overline{3}$

$$100\,x = 7613'33\ldots$$
$$-10\,x = 761'33\ldots$$
$$90\,x = 6852 \qquad x = \frac{6852}{90} = \boxed{\frac{1142}{15}}$$

b) $12'\overline{90} + 76'1\overline{3} = \frac{142}{11} + \frac{1142}{15} = \boxed{\frac{14692}{165}}$

c) $12'\overline{90} + 76'1\overline{3} = \boxed{89'0\overline{42}}$

② $M = 1'670 \cdot 10^{-24} \frac{\cancel{g}}{\cancel{at.H.}} \cdot 1'191 \cdot 10^{57}\ \cancel{at.H.} \cdot \frac{1}{1000} \frac{Kg}{\cancel{g}} = \boxed{1'989 \cdot 10^{30}\ Kg}$

③ $\left.\begin{array}{l} a=2 \\ b=3 \end{array}\right| \to a^2+b^2+(ab)^2 = 4+9+36 = 49 = 7^2$

$\left.\begin{array}{l} a=3 \\ b=4 \end{array}\right| \to a^2+b^2+(ab)^2 = 9+16+144 = 169 = 13^2$

Parece deducirse que $\left.\begin{array}{l} a \\ b=a+1 \end{array}\right| \to a^2+b^2+(ab)^2 = (a\cdot b+1)^2$

Demostraremos entonces que:

$$a^2 + (a+1)^2 + [a(a+1)]^2 = [a(a+1)+1]^2$$

$$a^2+(a+1)^2+[a(a+1)]^2 = a^2+a^2+2a+1+(a^2+a)^2 = 2a^2+2a+1+a^4+2a^3+a^2 = \boxed{a^4+2a^3+3a^2+2a+1}$$

$$[a(a+1)+1]^2 = a^2(a+1)^2 + 2a(a+1)+1 = a^2(a^2+2a+1)+2a^2+2a+1 = a^4+2a^3+a^2+2a^2+2a+1 = \boxed{a^4+2a^3+3a^2+2a+1}$$

④ Cuadrados: $1^2=1$; $2^2=4$; $3^2=9$; $4^2=16$; $5^2=25$; $6^2=36$; $7^2=49$; $8^2=64$; $9^2=81$; $10^2=100$

Cualquier otro número terminará en la cifra correspondiente al cuadrado de su última cifra. Por ejemplo 37^2 terminará en 9 (como 7^2).

Luego los cuadrados terminan en: $\boxed{0, 1, 4, 5, 6, 9}$

Cubos: $0^3=0$; $1^3=1$; $2^3=8$; $3^3=27$; $4^3=64$; $5^3=125$; $6^3=216$; $7^3=343$; $8^3=512$; $9^3=729$

Vemos que los cubos pueden terminar en $\boxed{\text{cualquier cifra}}$

⑤ $81 = \boxed{3^4}$ $\quad 0'5 = \boxed{2^{-1}}$ $\quad \frac{1}{25} = \boxed{5^{-2}}$ $\quad \sqrt[3]{9} = \boxed{3^{2/3}}$ $\quad \frac{1}{\sqrt[6]{8}} = 2^{-3/6} = \boxed{2^{-1/2}}$

⑥ $$\sqrt[3]{36} \cdot \sqrt[4]{24} = \sqrt[3]{3^2\cdot 2^2} \cdot \sqrt[4]{2^3\cdot 3} = (3^2\cdot 2^2)^{1/3} \cdot (2^3\cdot 3)^{1/4} = (3^2\cdot 2^2)^{4/12} \cdot (2^3\cdot 3)^{3/12} =$$
$$= \sqrt[12]{(3^2\cdot 2^2)^4\cdot(2^3\cdot 3)^3} = \sqrt[12]{3^8\cdot 2^8\cdot 2^9\cdot 3^3} = \sqrt[12]{2^{17}\cdot 3^{11}}$$

$$\boxed{\begin{array}{l} m=17 \\ n=12 \\ p=11 \end{array}}$$

(7) a) $\dfrac{81^{4/3}\cdot\sqrt{27}\cdot\sqrt{3\sqrt{3}}}{3^5\cdot 9^{-2}} = \dfrac{3^{8/3}\cdot 3^{3/2}\cdot 3^{3/4}}{3^5\cdot 3^{-4}} = 3^{\frac{8}{3}+\frac{3}{2}+\frac{3}{4}-5+4} =$

$\sqrt{3\sqrt{3}} = \sqrt{\sqrt{27}} = \sqrt[4]{27} = 3^{3/4}$

$= 3^{47/12} = \boxed{\sqrt[12]{3^{47}}} = \boxed{3^3\sqrt[12]{3^{11}}}$

b) $\sqrt{5}+\sqrt{45}+\sqrt{180}-\sqrt{80} = \sqrt{5}+3\sqrt{5}+6\sqrt{5}-4\sqrt{5} = (1+3+6-4)\sqrt{5} = \boxed{6\sqrt{5}} = \boxed{\sqrt{180}}$

c) $\sqrt[3]{\dfrac{3\sqrt{512}+5\sqrt{32}}{\sqrt{50}-\sqrt{18}}} = \sqrt[3]{\dfrac{48\sqrt{2}+20\sqrt{2}}{5\sqrt{2}-3\sqrt{2}}} = \sqrt[3]{\dfrac{68\sqrt{2}}{2\cancel{\sqrt{2}}}} = \boxed{\sqrt[3]{34}}$

d) $\dfrac{\sqrt[4]{a}\cdot a^3\, a^{1/2}}{a^{-3}\cdot\sqrt[3]{a}} = a^{\frac{1}{4}+3+\frac{1}{2}+3-\frac{1}{3}} = a^{77/12} = \boxed{\sqrt[12]{a^{77}}} = \boxed{a^6\sqrt[12]{a^5}}$

(8) $\left[\sqrt{2}+\sqrt{3}\right]^2 = 2+3+2\sqrt{6} = 5+2\sqrt{6}$

$\left(\sqrt{5+\sqrt{24}}\right)^2 = 5+\sqrt{24} = 5+2\sqrt{6}$

(9) a) $\dfrac{3}{\sqrt{6}} = \dfrac{3\sqrt{6}}{\sqrt{6}\sqrt{6}} = \dfrac{3\sqrt{6}}{6} = \boxed{\dfrac{\sqrt{6}}{2}}$

b) $\dfrac{6}{\sqrt[3]{2}} = \dfrac{6\sqrt[3]{4}}{\sqrt[3]{2}\sqrt[3]{4}} = \dfrac{6\sqrt[3]{4}}{2} = \boxed{3\sqrt[3]{4}} = \boxed{\sqrt[3]{108}}$

c) $\dfrac{\sqrt{27}+\sqrt{12}}{\sqrt{12}-\sqrt{3}} = \dfrac{(\sqrt{27}+\sqrt{12})(\sqrt{12}+\sqrt{3})}{(\sqrt{12}-\sqrt{3})(\sqrt{12}+\sqrt{3})} = \dfrac{\sqrt{324}+\sqrt{81}+\sqrt{144}+\sqrt{36}}{12-3} = \dfrac{18+9+12+6}{9} = \dfrac{45}{9} = \boxed{5}$

otra forma: $\dfrac{\sqrt{27}+\sqrt{12}}{\sqrt{12}-\sqrt{3}} = \dfrac{3\sqrt{3}+2\sqrt{3}}{2\sqrt{3}-\sqrt{3}} = \dfrac{5\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = 5$ ✓

d) $\dfrac{6}{\sqrt{7}+\sqrt{3}} = \dfrac{6\cdot(\sqrt{7}-\sqrt{3})}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \dfrac{6(\sqrt{7}-\sqrt{3})}{7-3} = \dfrac{6(\sqrt{7}-\sqrt{3})}{4} = \boxed{\dfrac{3(\sqrt{7}-\sqrt{3})}{2}}$

(10) a) $\sqrt{\dfrac{5}{18}}+3\sqrt{\dfrac{1}{8}} = \dfrac{\sqrt{5}}{\sqrt{18}}+\dfrac{3}{\sqrt{8}} = \dfrac{\sqrt{5}}{3\sqrt{2}}+\dfrac{3}{2\sqrt{2}} = \dfrac{\sqrt{5}\sqrt{2}}{3\sqrt{2}\sqrt{2}}+\dfrac{3\sqrt{2}}{2\sqrt{2}\sqrt{2}} =$

$= \dfrac{\sqrt{10}}{6}+\dfrac{3\sqrt{2}}{4} = \boxed{\dfrac{2\sqrt{10}+9\sqrt{2}}{12}} = \boxed{\dfrac{\sqrt{2}(2\sqrt{5}+9)}{12}}$

b) $\sqrt[4]{9}+\sqrt{\dfrac{1}{3}}-\sqrt{\dfrac{4}{27}} = 3^{2/4}+3^{-1/2}-2\cdot 3^{-3/2} = 3^{1/2}+3^{-1/2}-2\cdot 3^{-3/2} =$

$= \sqrt{3}+\dfrac{1}{\sqrt{3}}-\dfrac{2}{3\sqrt{3}} = \sqrt{3}+\dfrac{\sqrt{3}}{3}-\dfrac{2\sqrt{3}}{9} = \dfrac{9\sqrt{3}+3\sqrt{3}-2\sqrt{3}}{9} = \boxed{\dfrac{10\sqrt{3}}{9}}$