① a)

| | | | |
|---|---|---|---|
| $x^5$ | $+7x^3$ | $-5x+1$ | $\mid x^3+2x$ |
| $-x^5$ | $-2x^3$ | | $x^2+5$ |
| | $5x^3$ | $-5x+1$ | |
| | $-5x^2$ | $-10x$ | |
| | | $-15x+1$ | |

Cociente $= x^2+5$
Resto $= -15x+1$

b)

| | | | |
|---|---|---|---|
| $-2x^4+5x^3$ | $-x+4$ | | $\mid x^2-3$ |
| $2x^4$ | $-6x^2$ | | $-2x^2+5x-6$ |
| | $5x^3-6x^2-x+4$ | | |
| | $-5x^3$ | $+15x$ | |
| | | $-6x^2+14x+4$ | |
| | | $6x^2$ $-18$ | |
| | | $14x-14$ | |

Cociente $= -2x^2+5x-6$
Resto $= 14x-14$

c)

| | | | | | |
|---|---|---|---|---|---|
| $-1$ | $-1$ | $2$ | $0$ | $-3$ | $1$ |
| | | $1$ | $-3$ | $3$ | $0$ |
| | $-1$ | $3$ | $-3$ | $0$ | $1$ |

Cociente $= -x^3+3x^2-3x$
Resto $= 1$

② $(x^3+2x)(x^2+5)+(-15x+1) = x^5+5x^3+2x^3+10x-15x+1 = x^5+7x^3-5x+1$ ✓

③

| | | | | |
|---|---|---|---|---|
| $-2$ | $4$ | $-k$ | $2$ | $-7$ |
| | | $-8$ | $2k+16$ | $-4k-36$ |
| | $4$ | $-k-8$ | $2k+18$ | $-4k-43$ |

$-4k-43=1$ ; $\boxed{k=-11}$

$4\cdot(-2)^3-k\cdot(-2)^2+2\cdot(-2)-7=1$
$-32-4k-4-7=1$
$-4k=44$ ; $\boxed{k=-11}$

④ $2(x-2)(x-1)(x+1) = (2x-4)(x^2-1) = \boxed{2x^3-4x^2-2x+4}$

⑤ $(x-3)(x+1) = x^2-2x-3$ $\quad \boxed{x^2-2x-3=0}$

⑥ a) $x^2-4 = \boxed{(x+2)(x-2)}$ b) $9x^2-4 = \boxed{(3x+2)(3x-2)}$ c) $4x^4-x^2 = \boxed{x^2(4x^2-1) = x^2(2x+1)(2x-1)}$

d) $12x-3x^3 = 3x(4-x^2) = \boxed{3x(2+x)(2-x)}$ e) $x^2+2x+1 = \boxed{(x+1)^2}$ f) $x^2-4x+4 = \boxed{(x-2)^2}$

g) $x^4-18x^2+81 = (x^2-9)^2 = \boxed{(x+3)^2(x-3)^2}$ h) $12x^3+12x^2+3x = 3x(4x^2+4x+1) = \boxed{3x(2x+1)^2}$

⑦ a)

| | | | | | |
|---|---|---|---|---|---|
| $1$ | $1$ | $-2$ | $-3$ | $8$ | $-4$ |
| | | $1$ | $-1$ | $-4$ | $4$ |
| $1$ | $1$ | $-1$ | $-4$ | $4$ | $0$ |
| | | $1$ | $0$ | $-4$ | |
| $2$ | $1$ | $0$ | $-4$ | $0$ | |
| | | $2$ | $4$ | | |
| $-2$ | $1$ | $2$ | $0$ | | |
| | | $-2$ | | | |
| | $1$ | $0$ | | | |

$\boxed{(x-1)^2(x+2)(x-2)} = x^4-2x^3-3x^2+8x-4$

b) $2x^5-6x^3-4x^2 = 2x^2(x^3-3x-2) =$

| | | | | |
|---|---|---|---|---|
| $2$ | $1$ | $0$ | $-3$ | $-2$ |
| | | $2$ | $4$ | $2$ |
| $-1$ | $1$ | $2$ | $1$ | $0$ |
| | | $-1$ | $-1$ | |
| $-1$ | $1$ | $1$ | $0$ | |
| | | $-1$ | | |
| | $1$ | $0$ | | |

$= \boxed{2x^2\cdot(x-2)(x+1)^2}$

⑧ a)

| | | | | |
|---|---|---|---|---|
| $1$ | $1$ | $-12$ | $41$ | $-30$ |
| | | $1$ | $-11$ | $30$ |
| $5$ | $1$ | $-11$ | $30$ | $0$ |
| | | $5$ | $-30$ | |
| $6$ | $1$ | $-6$ | $0$ | |
| | | $6$ | | |
| | $1$ | $0$ | | |

$\boxed{x=\begin{cases}1\\5\\6\end{cases}}$

b)

| | | | | | |
|---|---|---|---|---|---|
| $2$ | $1$ | $-4$ | $-1$ | $16$ | $-12$ |
| | | $2$ | $-4$ | $-10$ | $12$ |
| $1$ | $1$ | $-2$ | $-5$ | $6$ | $0$ |
| | | $1$ | $-1$ | $-6$ | |
| $3$ | $1$ | $-1$ | $-6$ | $0$ | |
| | | $3$ | $6$ | | |
| $-2$ | $1$ | $2$ | $0$ | | |
| | | $-2$ | | | |
| | $1$ | $0$ | | | |

$\boxed{x=\begin{cases}1\\2\\-2\\3\end{cases}}$

(9) a) $(2x+1)^5 = (2x)^5 + 5\cdot(2x)^4\cdot 1 + 10\cdot(2x)^3\cdot 1^2 + 10\,(2x)^2\cdot 1^3 + 5\cdot(2x)\cdot 1^4 + 1^5 =$

$= \boxed{32x^5 + 80x^4 + 80x^3 + 40x^2 + 10x + 1}$

b) $(1-3x)^4 = 1 - 4\cdot 3x + 6\cdot(3x)^2 - 4\cdot(3x)^3 + (3x)^4 =$

$= \boxed{1 - 12x + 54x^2 - 108x^3 + 81x^4}$

c) $(\sqrt{2}-1)^3 = (\sqrt{2})^3 - 3\cdot(\sqrt{2})^2 + 3\cdot\sqrt{2} - 1 = 2\sqrt{2} - 6 + 3\sqrt{2} - 1 = \boxed{5\sqrt{2} - 7}$

(10) a) $\frac{x+2}{x^2-9}\cdot\frac{2x+6}{3x+6} - \frac{1}{2}\,\frac{1-x}{x-3} = \frac{(x+2)\cdot 2(x+3)}{(x+3)(x-3)\cdot 3(x+3)} - \frac{1-x}{2(x-3)} = \frac{2}{3(x-3)} - \frac{1-x}{2(x-3)} =$

$= \frac{4 - 3 + 3x}{6(x-3)} = \boxed{\frac{3x+1}{6(x-3)}}$

b) $\frac{6}{1-x} - \frac{x}{x^2-x} = \frac{6}{1-x} - \frac{1}{x-1} = \frac{-6}{x-1} - \frac{1}{x-1} = \boxed{\frac{-7}{x-1}}$

c) $\frac{4}{3x^2-3x-18} - \frac{3}{2x+4} = \frac{4}{3(x^2-x-6)} - \frac{3}{2(x+2)} = \frac{4}{3(x+2)(x-3)} - \frac{3}{2(x+2)} =$

$= \frac{8 - 3\cdot 3(x-3)}{6(x+2)(x-3)} = \frac{8 - 9x + 27}{6(x+2)(x-3)} = \boxed{\frac{35 - 9x}{6(x+2)(x-3)}}$

(11) a) $\frac{6x-12}{4} = x - 5$ ; $6x - 12 = 4x - 20$ ; $2x = -8$ ; $\boxed{x = -4}$

b) $4(x-9) - x + 5 = 4x + 20$ ; $4x = 24$ ; $\boxed{x = 6}$

c) $\frac{6}{8} - \frac{12x-18}{16} - \frac{9x}{4} = \frac{9x-6}{8}$ ; $12 - 12x + 18 - 36x = 18x - 12$ ; $-66x = -42$ ; $\boxed{x = \frac{7}{11}}$

(12) a) $x(3x+2) = 0$ ; $\boxed{x = \begin{cases} 0 \\ -2/3 \end{cases}}$ b) $x^2 = 4$ ; $\boxed{x = \pm 2}$

c) $3x^2 - 9x - 30 = 0$ ; $x^2 - 3x - 10 = 0$ ; $\boxed{x = \begin{cases} 5 \\ -2 \end{cases}}$

d) $10x^2 + 15x - 8x - 12 = 5$ ; $10x^2 + 7x - 17 = 0$ ; $\boxed{x = \begin{cases} 1 \\ -17/10 \end{cases}}$

(13) a) $(2-x)(x+2) + 8 = 2(x+2)$ ; $4 - x^2 + 8 = 2x + 4$ ; $0 = x^2 + 2x - 8$ ; $\boxed{x = \begin{cases} 2 \\ -4 \end{cases}}$

b) $\frac{x^2 - 3x + 2}{2} = 1 - \frac{2x - x^2}{3}$ ; $3x^2 - 9x + 6 = 6 - 6x + 2x^2$ ; $x^2 - 5x = 0$ ; $\boxed{x = \begin{cases} 0 \\ 5 \end{cases}}$

c) $2(x+1)^2 - (x+1) = 36$ ; $2x^2 + 4x + 2 - x - 1 - 36 = 0$ ; $2x^2 + 3x - 35 = 0$

$x = \frac{-3 \pm \sqrt{9 + 280}}{4} = \frac{-3 \pm 17}{4} \begin{cases} 14/4 = \frac{7}{2} \\ -\frac{20}{4} = -5 \end{cases}$

d) $x^2 - 9 = 3 - x$ ; $x^2 + x - 12 = 0$ $\boxed{x = \begin{cases} -4 \\ 3 \end{cases}}$

(14) a) $x^2(x^2-9)=0$ ; $\boxed{\begin{matrix}x=0\\ \pm 3\end{matrix}}$

b) $x^4-13x^2+36=0$

$x^2=\begin{cases}9 \to \boxed{x=\pm 3}\\ 4 \to \boxed{x=\pm 2}\end{cases}$

c) $x^4+x^2=2$ ; $x^4+x^2-2=0$ ; $x^2=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm 3}{2}=\begin{cases}1 \to x=\pm\sqrt{1}=\boxed{\pm 1}\\ -2 \to x=\pm\sqrt{-2}\ \times\end{cases}$

(15) a) $3\sqrt{x-1}=2x-4$ ; $9(x-1)=4x^2-16x+16$ ; $0=4x^2-25x+25$ ;

$x=\frac{25\pm\sqrt{625-400}}{8}=\frac{25\pm 15}{8}\begin{cases}40/8=\boxed{5} \to \sqrt{5-1}=2\cdot 5-4\ \checkmark\\ 5/4 \to 3\sqrt{\frac{5}{4}-1}\ \not=\ \frac{10}{4}-4\end{cases}$

b) $\sqrt{x+5}=3-\sqrt{x+2}$

$x+5=9-6\sqrt{x+2}+x+2$ ; $6\sqrt{x+2}=6$ ; $\sqrt{x+2}=1$ ; $x+2=1$ ; $\boxed{x=-1}$

$\sqrt{-1+5}=3-\sqrt{-1+2}$ ✓

c) $\sqrt{2x+1}=1+\sqrt{x}$

$2x+1=1+2\sqrt{x}+x$ ; $x=2\sqrt{x}$ ; $x^2=4x$ ; $x^2-4x=0$ ; $x(x-4)=0$ → $\boxed{x=0}$ $\sqrt{0+1}=1+\sqrt{0}$ ✓; $\boxed{x=4}$ $\sqrt{8+1}=1+\sqrt{4}$ ✓

(16) a) $\begin{cases}2x+3y=3\\ 5x+7y=8\end{cases}$ $\begin{cases}10x+15y=15\\ -10x-14y=-16\end{cases}$

$\boxed{y=-1}$ → $\boxed{x=3}$

b) $\begin{cases}x+\frac{6}{y}=2\\ 3x-y=-5\end{cases}$ → $y=(3x+5)$

$x+\frac{6}{3x+5}=2$ ; $3x^2+5x+6=6x+10$

$3x^2-x-4=0$

$x=\frac{1\pm\sqrt{1+48}}{6}=\frac{1\pm 7}{6}\begin{cases}4/3\\ -1\end{cases}$

$\boxed{x=\frac{4}{3} \to y=9}$

$\boxed{x=-1 \to y=2}$

c) $\begin{cases}x^2+y=3\\ 2x-y=5\end{cases}$ → $y=(2x-5)$

$x^2+2x-5=3$ ; $x^2+2x-8=0$ ; $x=\frac{-2\pm\sqrt{4+32}}{2}=\frac{-2\pm 6}{2}\begin{cases}2\\ -4\end{cases}$

$\boxed{x=2 \to y=-1}$

$\boxed{x=-4 \to y=-13}$

(17) $\begin{cases}(-2)^2+b\cdot(-2)+c=0\\ (-1)^2+b\cdot(-1)+c=-4\end{cases}$ $\begin{cases}-2b+c=-4\\ -b+c=-5\end{cases}$

$-b=1$ $\boxed{b=-1}$ → $\boxed{c=-6}$

(18) $\begin{cases}x-y+3z=4\\ 2x-y+2z=6\\ -3x+z=-8\end{cases}$ $\xrightarrow{\begin{matrix}E2=E2-2E1\\ E3=E3+3E1\end{matrix}}$ $\begin{cases}x-y+3z=4\\ +y-4z=-2\\ -3y+10z=4\end{cases}$ $\xrightarrow{E3\to E3+3E2}$ $\begin{cases}x-y+3z=4\\ y-4z=-2\\ -2z=-2\end{cases}$

$\boxed{z=1}$ → $\boxed{y=2}$ → $\boxed{x=3}$

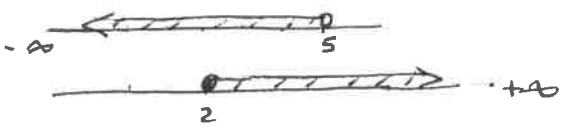
(19) a] $\frac{1}{3}(2-x) \leq x+2$

$\frac{2-x}{3} \leq x+2$ ; $2-x \leq 3x+6$ ; $-4x \leq 4$ ; $x \geq \frac{4}{-4}$ ; $x \geq -1$ $\quad \boxed{x \in [-1, +\infty)}$

b] $\begin{cases} 1 - \frac{x-3}{2} > x-5 \\ 3x-2 \geq 4-(2-x) \end{cases}$

$\rightarrow 2-(x-3) > 2x-10$ ; $2-x+3 > 2x-10$ ; $-3x > -15$ ; $x < \frac{-15}{-3}$ ; $x < 5$

$\rightarrow 3x-2 \geq 4-2+x$ ; $2x \geq 4$ ; $x \geq 2$

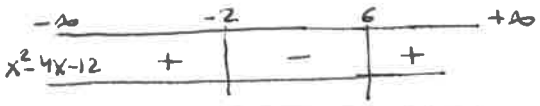



$\boxed{x \in [2,5)}$

c] $x^2 - 4x - 12 > 0$

$x^2 - 4x - 12 = 0$

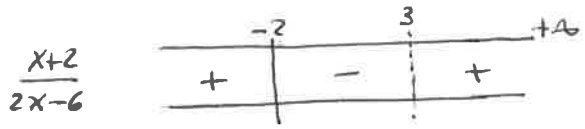
$x = \frac{4 \pm \sqrt{16+48}}{2} = \frac{4 \pm 8}{2} \begin{cases} 6 \\ -2 \end{cases}$




$\boxed{x \in (-\infty, -2) \cup (6, +\infty)}$

d] $\frac{x+2}{2x-6} \leq 0$

$\begin{cases} 2x-6 \neq 0 \\ \frac{x+2}{2x-6} = 0 \text{ ; } x+2=0 \text{ ; } x=-2 \\ \rightarrow 2x-6=0 \text{ ; } x=3 \end{cases}$




$\boxed{x \in [-2, 3)}$

(20) $3x^2 + 2bx + 3 = 0$

$x = \frac{-2b \pm \sqrt{4b^2-36}}{6}$

Solución única $\Rightarrow$ $4b^2 - 36 = 0$ ; $b^2 = 9$ ; $b = \begin{cases} \boxed{3} \\ \cancel{-3} \end{cases}$ Dicen que $b > 0$

(21) $2x^2 - bx + 8 = 0$

$x = \frac{b \pm \sqrt{b^2-64}}{4}$

Dos soluciones $\Rightarrow$ $b^2 - 64 > 0$ ; $b^2 > 64$ ; $b > 8$ ó $b < -8$ ; $\boxed{b \in (-\infty, -8) \cup (8, +\infty)}$

(22) $\left(2x^2 - \frac{3}{x}\right)^6 = \sum_{r=0}^{6} \binom{6}{r} (2x^2)^r \cdot \left(\frac{-3}{x}\right)^{6-r} = \sum_{r=0}^{6} \binom{6}{r} \frac{2^r \cdot x^{2r} \cdot (-3)^{6-r}}{x^{6-r}}$

$2r = 6-r$ ; $3r = 6$ ; $r = 2$ ; $\binom{6}{2} \cdot \frac{2^2 \cdot x^4 \cdot (-3)^4}{x^4} = 15 \cdot 4 \cdot 81 = \boxed{4860}$

(23) $(2+ax)^{10} = \sum_{r=0}^{10} \binom{10}{r} 2^r \cdot (ax)^{10-r}$

$10-r = 3 \rightarrow r = 7$ $\quad \binom{10}{7} \cdot 2^7 \cdot a^3 \cdot x^3 = 15360 a^3 x^3$

$15360a^3 = 414720$ ; $a^3 = 27$ ; $a = \sqrt[3]{27} = \boxed{3}$

30) $x$ = nº vecinos
$p$ = precio obra

$$\left.\begin{array}{l} 240 \cdot x = p \\ 300(x-3) = p \end{array}\right\} \rightarrow \begin{array}{l} 240x = 300x - 900 \\ 900 = 60x \\ \boxed{15 = x} \end{array}$$

15 vecinos

p obra $\rightarrow$ $p = 240 \cdot 15 = \boxed{3600\ €}$ precio obra.

(32) $x \rightarrow$ nº kg patatas
$y \rightarrow$ nº kg manzanas
$z \rightarrow$ nº kg naranjas.

$$\left.\begin{array}{l} x + y + z = 9 \\ x + 1'20y + 1'50z = 11'60 \\ z = y + 1 \end{array}\right\}$$

$$\left.\begin{array}{l} x + y + z = 9 \\ 100x + 120y + 150z = 1160 \\ -y + z = 1 \end{array}\right\} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 100 & 120 & 150 & 1160 \\ 0 & -1 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 20 & 50 & 260 \\ 0 & -1 & 1 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & 1 & 1 \\ 0 & 20 & 50 & 260 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 70 & 280 \end{array}\right) \Rightarrow \left.\begin{array}{l} x + y + z = 9 \\ -y + z = 1 \\ 70z = 280 \end{array}\right\}$$

$z = \frac{280}{70} = 4$ Kg de naranjas; $y = z - 1 = 4 - 1 = 3$ Kg de manzanas.

$x = 9 - y - z = 9 - 3 - 4 = 2$ Kg de patatas

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(34) $x \rightarrow$ nº hombres
$y \rightarrow$ nº mujeres
$z \rightarrow$ nº niños

$$\left.\begin{array}{l} x + y + z = 20 \\ x + y = 3z \\ y + 1 = x \end{array}\right\} \quad \left.\begin{array}{l} x + y + z = 20 \\ x + y - 3z = 0 \\ -x + y = -1 \end{array}\right\}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 1 & 1 & -3 & 0 \\ -1 & 1 & 0 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 0 & -4 & -20 \\ 0 & 2 & 1 & 19 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 2 & 1 & 19 \\ 0 & 0 & -4 & -20 \end{array}\right)$$

$$\left.\begin{array}{l} x + y + z = 20 \\ 2y + z = 19 \\ -4z = -20 \end{array}\right\} \quad \boxed{z = 5} \rightarrow 2y + 5 = 19 \rightarrow 2y = 14 \rightarrow \boxed{y = 7}$$

$$x = 20 - y - z = 20 - 7 - 5 = \boxed{8 = x}$$

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(36) $x \to$ nº cajas de 250 g.
$y \to$ nº cajas de 500 g.
$z \to$ nº cajas de 1 Kg

$$\left.\begin{array}{l} x+y+z=60 \\ x=y+5 \\ 0'25x+0'5y+z=\frac{1250}{40} \end{array}\right\}$$

$$\left.\begin{array}{l} x+y+z=60 \\ x-y=5 \\ 25x+50y+100z=3125 \end{array}\right\} \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 1 & -1 & 0 & 5 \\ 25 & 50 & 100 & 3125 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & -2 & -1 & -55 \\ 0 & 25 & 75 & 1625 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & -2 & -1 & -55 \\ 0 & 0 & 125 & 1875 \end{array}\right) \to \left.\begin{array}{l} x+y+z=60 \\ -2y-z=-55 \\ 125z=1875 \end{array}\right\}$$

$\boxed{z=15}$

$-2y=-55+15 \to \boxed{y=20}$

$x=60-y-z=\boxed{25=x}$

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o

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