SUCESIONES Y SERIES

1 $a_1 = -7$
$S_{20} = 620$

a) $S_{20} = \dfrac{(a_1 + a_{20}) \cdot 20}{2}$

$620 = \dfrac{((-7) + a_{20}) \cdot 20}{2}$; $\boxed{a_{20} = 69}$

~~b)~~ $a_{20} = a_1 + (n-1) \cdot d$

$69 = -7 + (19) \cdot d \longrightarrow \boxed{d = 4}$

b) $a_{78} = a_1 + (77) \cdot 4 \longrightarrow a_{78} = -7 + 308 = \boxed{301}$

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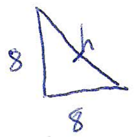
2 a)

n	1	2	3
x_n	8	$\boxed{4\sqrt{2}}$	4
A_n	32	16	$\boxed{8}$



$\rightarrow 2 \cdot 64 = h^2$

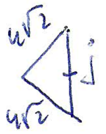
$\sqrt{128} = h$

$\sqrt{2^7} = h = 2^3 \cdot \sqrt{2} = \boxed{8\sqrt{2}}$

luego $\boxed{4\sqrt{2}}$ ✓

$j^2 = (4\sqrt{2})^2 + (4\sqrt{2})^2 = 64$

$j = 8$.



luego el lado del siguiente es $\underline{4}$

$A_1 = \dfrac{8 \cdot 8}{2} = 32$

$A_2 = \dfrac{4\sqrt{2} \cdot 4\sqrt{2}}{2} = 16$

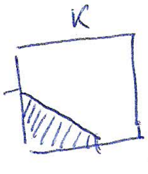
$A_3 = \dfrac{4 \cdot 4}{2} = 8$

b) $A_1 = 32$
$A_2 = 16$
$A_3 = 8$

$A_n = 32 \cdot \left(\frac{1}{2}\right)^{n-1}$

$A_6 = 32 \cdot \left(\frac{1}{2}\right)^5 = \boxed{1}$

c) $A_n = \dfrac{K^2}{8} \cdot \left(\dfrac{1}{2}\right)^{n-1}$



$A_1 = \dfrac{K/2 \cdot K/2}{2} = \dfrac{K^2}{8}$

Si $S_n = K$

$S_n = \dfrac{a_1}{1-r}$; $|r| < 1 \Longrightarrow K = \dfrac{\frac{K^2}{8}}{1 - \frac{1}{2}}$; $K = \dfrac{\frac{K^2}{8}}{\frac{1}{2}}$; $K = \dfrac{2K^2}{8}$;

$8K = 2K^2$; $2K^2 - 8K = 0$; $2K(K-4)$ $\boxed{K=0}$ ✗ $\boxed{K=4}$ ✓

3) $S_1 = 1+K$

a) $S_2 = 5+3K \longrightarrow u_1 + u_2 \longrightarrow 5+3K = (1+K) + u_2$

$\boxed{u_1 = 1+K}$ $\qquad$ $\boxed{4+2K = u_2}$

$S_3 = S_2 + u_3 \longrightarrow 12+7K = (5+3K) + u_3$

$\boxed{7+4K = u_3}$

$S_4 = S_3 + u_4 \longrightarrow 22+15K = (12+7K) + u_4$

$\boxed{10+8K = u_4}$

b) $\boxed{u_n = b + cK}$

$1, 4, 7, 10 \ldots \quad d=3$

$b_n = b_1 + (n-1)\cdot d$

$b_n = 1 + (n-1)\cdot 3$

$b_n = 1 + 3n - 3 = \boxed{3n-2}$

$1K, 2K, 4K, 8K \ldots$

$2^0K \quad 2^1K, \quad 2^2K, \quad 2^3K, \ldots 2^{n-1}K$

$\boxed{u_n = 3n-2 + 2^{n-1}K}$

– Ej para $n=2 \rightarrow u_2 = 4+2K$

$n=3 \rightarrow u_3 = 7+4K$

$n=1 \rightarrow u_1 = 1+K$

⋮

4) a) $\left.\begin{array}{l} a_1 = 2 \\ a_2 = 5 \end{array}\right\} \rightarrow d = a_2 - a_1 = \boxed{3}$

b) $a_n = a_1 + (n-1)\cdot d$

$a_8 = 2 + 7\cdot 3 = \boxed{23}$

c) $S_8 = \dfrac{(a_1+a_8)\cdot 8}{2} = \dfrac{(2+23)\cdot 8}{2} = \boxed{100}$

5) a) i) $\left.\begin{array}{l} u_1 = 4 \\ u_2 = 4{,}2 \end{array}\right\} \; r = \dfrac{u_2}{u_1} = \dfrac{4'2}{4} = \boxed{1'05}$

ii) $u_5 = u_1 \cdot r^{5-1}$

$u_5 = 4 \cdot 1'05^4 = \boxed{4{,}862025}$ $\quad \boxed{4{,}86}$

5 cont. $V_n = a\cdot n^k$

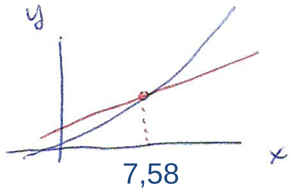
b) i) $V_1 \longrightarrow 0'05 = a\cdot 1^k \longrightarrow \boxed{0'05 = a}$

ii) $V_2 \longrightarrow 0'25 = 0'05\cdot 2^k \longrightarrow 5 = 2^k \longrightarrow K = \log_2 5 = \frac{\log 5}{\log 2} = \boxed{2'321928}$ $\boxed{2'32}$

c) $V_n > u_n \longrightarrow \overset{0'05}{a}\cdot n^k >$ 4 x 1,05^(n-1)

$0'05\cdot n^k >$ 4 x 1,05^(n-1)

$0'05\cdot n^{2'321928} >$ 4 x 1,05^(n-1)



Con $n =$ 7,58? los funcion se igualan

con $\boxed{n = 8}$ la función $0'05\cdot n^{2'321...}$ es mayor q la otra

↓

menor valor de n. por tanto.

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6 a) $u_{11} - u_{10} = d \longrightarrow 6'5 - 8 = \boxed{-1'5} = d.$

b) $u_{11} = u_1 + 10\cdot(-1'5)$

$6,5 = u_1 - 15 \longrightarrow \boxed{u_1 = 21'5}$

c) $S_{50} = \frac{(u_1 + u_{50})\cdot 50}{2} = \frac{(21'5 - 52)\cdot 50}{2} = \boxed{-762'5}$

$u_{50} = \overset{21'5}{u_1} + 49\cdot(-1'5) =$

$u_{50} = -52$

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7 $u_1 = 80$

$u_2 = 80\cdot 0'9$

$u_3 = 80\cdot 0'9^2$

$u_n = 80\cdot(0'9)^{n-1}$

$r = 0'9 \rightarrow < 1$

$S_{15} = \frac{u_1(r^{15}-1)}{r-1} = \frac{80(0'9^{15}-1)}{0'9-1} = $ 635'29 m recorre en 15'

menos q 660 m. No llega

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8 $a_1 = \ln a$

$d = \ln 3$

$a_{13} = 8\ln 9$

$a_n = a_1 + (n-1)\cdot d$

$a_{13} = \ln a + 12\cdot \ln 3$

$8\ln 9 = \ln a\cdot 3^{12}$

$\ln 9^8 = \ln a\cdot 3^{12}$

$9^8 = a\cdot 3^{12}$

$(3^2)^8 = a\cdot 3^{12}$

$3^{16} = a\cdot 3^{12}$

$\frac{3^{16}}{3^{12}} = a = 3^4 = \boxed{81}$

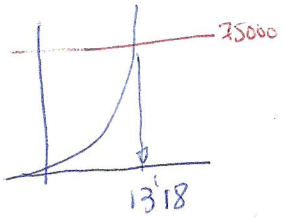
$\boxed{a = 81}$

9

a) $\boxed{r = \dfrac{u_2}{u_1} = \dfrac{1.6}{0'64} = 2'5}$

b) $S_6 = \dfrac{u_1 \cdot (r^6 - 1)}{r-1} = \dfrac{0'64 \cdot (2'5^6 - 1)}{2'5 - 1} = \boxed{103'74}$

c) $75000 = \dfrac{0'64 \cdot (2'5^n - 1)}{2'5 - 1} \longrightarrow$ con CG



Con $n = 13'18$ la suma es exactamente 75000

Con $\boxed{n = 14}$ la suma es > 75.000

10

$5, 9, 13 \ldots$

a) $u_n = u_1 + (n-1) \cdot d \longrightarrow \boxed{u_n = 5 + (n-1) \cdot 4}$

$u_{200} = 5 + 199 \cdot 4 = \boxed{801}$ efectivamente 801 siguientes c.q.d.

b) $S_{200} = \dfrac{(u_1 + u_{200}) \cdot 200}{2} = \dfrac{(5 + 801) \cdot 200}{2} = \boxed{80.600}$

11

$r = \dfrac{6}{x-3}$, $r = \dfrac{x+2}{6}$

$\dfrac{6}{x-3} = \dfrac{x+2}{6} \Rightarrow 36 = x^2 + 2x - 3x - 6 \Rightarrow \cancel{x^2 + 2x}$

$x^2 - x - 42 = 0$

$x = \dfrac{1 \pm \sqrt{1 + 4 \cdot 42}}{2} = \dfrac{1 \pm \sqrt{1 + 168}}{2} = \dfrac{1 \pm 13}{2}$ → $\boxed{x_1 = 7}$, $\boxed{x_2 = -6}$

posibles valores.

12

$u_4 = 8 \cdot u_1 \longrightarrow u_4 = 2^3 \cdot u_1 \Rightarrow \boxed{2 = r}$

$S_{10} = \dfrac{u_1 (2^{10} - 1)}{2 - 1} \longrightarrow 2557.5 = \dfrac{u_1 \cdot (2^{10} - 1)}{1}$; $u_1 = \dfrac{2557.1}{(2^{10} - 1)} = \boxed{2,4996089}$

$\boxed{u_1 = 2,50}$

$u_{10} = u_1 \cdot r^9 = 2'4996089 \cdot 2^9 = \boxed{1279'7998} \rightarrow \boxed{1280}$ usando $\boxed{u_1 = 2'50}$

$\boxed{12'79'80} \rightarrow \boxed{1280}$

13

a) $d = u_2 - u_1 = 1'5 - 0'3 = \boxed{1'2}$

b) $u_{30} = 0'3 + 29 \cdot 1'2 = \boxed{35'1}$

c) $S_{30} = \dfrac{(0'3 + 35'1) \cdot 30}{2} = \boxed{531}$

14

a) $r = \frac{u_2}{u_1} = \frac{\cancel{\log_2 x}}{2 \cdot \cancel{\log_2 x}} = \boxed{\frac{1}{2}}$

b) $S_\infty = \frac{u_1}{1-r}$; $|r|<1 \longrightarrow S_\infty = \frac{2\log_2 x}{1-\frac{1}{2}} = \frac{2\log_2 x}{\frac{1}{2}} = \boxed{4\log_2 x}$ c.q.d.

c) $d = u_2 - u_1 = \log_2\left(\frac{x}{2}\right) - \log_2 x = \log_2 \frac{\frac{x}{2}}{2} = \boxed{\log_2 \frac{x}{4}}$ → No válido para expresar en $\mathbb{Z}$

probemos con:

$d = u_3 - u_2 = \log_2\left(\frac{x}{4}\right) - \log_2\left(\frac{x}{2}\right) = \log_2 \frac{\frac{x}{4}}{\frac{x}{2}} = \log_2 \frac{2\cancel{x}}{4\cancel{x}} = \log_2 \frac{1}{2} = \boxed{-1}$

d) $S_{12} = \frac{(u_1 + u_{12}) \cdot 12}{\cancel{2}}$

$u_n = u_1 + (n-1) \cdot d$

$u_{12} = \log_2 x + 11 \cdot (-1) = \boxed{\log_2 x - 11}$

$S_{12} = \frac{(\log_2 x + \log_2 x - 11) \cdot 12}{\cancel{2}} =$

$= (\log_2 x + \log_2 x - 11) \cdot \cancel{6} = (2\log_2 x - 11) \cdot 6 = \boxed{12\log_2 x - 66}$ c.q.d.

e) $12\log_2 x - 66 = \frac{4 \cdot \log_2 x}{2}$

$12\log_2 x - 66 = 2\log_2 x \longrightarrow 10\log_2 x = 66$; $\longrightarrow \log_2 x = 6'6$

$\boxed{2^{66} = x}$ $\quad 6'6 = \frac{66}{10} = \frac{33}{5}$; $\boxed{x = 2^{\frac{33}{5}}}$

15

a) $d = u_2 - u_1 = 7 - 3 = \boxed{4}$

b) $u_{10} = u_1 + (n-1) \cdot d \rightarrow u_{10} = 3 + (9) \cdot 4 = \boxed{39}$

c) $S_{10} = \frac{(u_1 + u_{10})10}{2} = \frac{(3+39)10}{2} = 42 \times 5 = \boxed{210}$

15 OTRO (el 2º)

a) i) $\boxed{p = 6}$ filas

ii) $\boxed{q = 5}$ column.

b) $6 \times 5 = \boxed{30}$ cm^2

c) $2, 6, 12, 20, 30 \ldots$

$12 = 4 \cdot 3$, $20 = 5 \cdot 4$, $30 = 6 \cdot 5$

nº fila x nº column. $\longrightarrow (n+1) \cdot n$

para figura n → (n+1)   n+1 = 5

para figura 4 es 4 → n.

expresión Area $\longrightarrow A_n = \boxed{(n+1) \cdot n} = \boxed{n^2 + n}$

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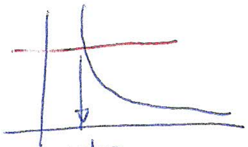
16 $r = \dfrac{576}{768} = 0'75$ $|r| < 1$

$a_n = a_1 \cdot r^{n-1} \longrightarrow 7 = 768 \cdot 0'75^{n-1} \xrightarrow{\text{Con C.G.}}$



Con $n = 17'33$ es exactamente 7

con $\boxed{n = 18}$ es < 7

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17 a) $r = \dfrac{u_2}{u_1} = \dfrac{\ln x^8}{\ln x^{16}} = \dfrac{8 \cancel{\ln x}}{16 \cancel{\ln x}} = \boxed{\dfrac{1}{2}}$

b) $\displaystyle\sum_{k=1}^{\infty} 2^{5-k} \cdot \ln x = 64 \longrightarrow S_\infty \left(2^{5-k} \cdot \ln x\right) = 64$

progresión geométrica

$u_1 = 2^4 \cdot \ln x = \ln x^{16}$

$u_2 = 2^3 \cdot \ln x = \ln x^8$

$\vdots$

Es la progresión geométrica inicial.

$S_\infty = \dfrac{u_1}{1-r}$, $|r| < 1$

$S_\infty = \dfrac{\ln x^{16}}{1 - \frac{1}{2}} = \dfrac{\ln x^{16}}{\frac{1}{2}} = 2 \ln x^{16} = \boxed{\ln x^{32}}$

$\ln x^{32} = 64$

$32 \ln x = 64 \longrightarrow \ln x = 2$; $\boxed{e^2 = x}$