1

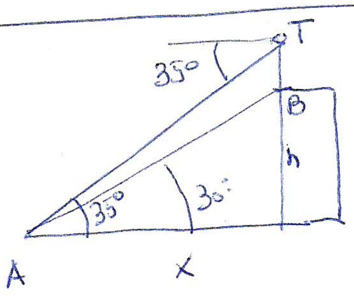
a) $6\sin x\cos x = 3\cdot 2\sin x\cdot\cos x = \boxed{3\sin 2x}$ $a=3$, $b=2$

b) $3\sin 2x = \frac{3}{2} \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = 30°$ ó $150°$

$\boxed{\cancel{x=15°}}$ No válido ó $\boxed{x=75°} = \boxed{\frac{5\pi}{12}}$

Se da en radianes, pues ellos lo dan en el

2



$tg\,30 = \frac{h}{x}$
$tg\,35 = \frac{h+1'6}{x}$

$\frac{\sqrt{3}}{3}\cdot x = h$

$0'7 = \frac{\sqrt{3}/3\cdot x + 1'6}{x}$; $0'7x = 0'5774x + 1'6$

$0'1226x = 1'6$

$\boxed{x \simeq 13'05\ m}$

$\boxed{h = 7'53}$ m.

Esto lo pondrán mal en el BI
El resultado es 7'52 m.

No se puede redondear hasta el último momento. (Y casi mejor no hacerlo).

~~3~~

Correcto: Despejar x, e igualarlo. Así no se redondea hasta el último moment.

$\cancel{h} = tg\,30$

$x = \frac{h}{tg\,30}$; $x = \frac{h+1'6}{tg\,35}$ $\Big\}$ $\frac{h}{tg\,30} = \frac{h+1'6}{tg\,35} \Rightarrow$

$h\cdot tg\,35 = h\cdot tg\,30 + 1'6\cdot tg\,30$

$h(tg\,35 - tg\,30) = 1'6\,tg\,30$

$h = \frac{1'6\cdot tg\,30}{(tg\,35 - tg\,30)}$ $\Rightarrow$ Hacerlo ahora con la calculadora de un tirón

$h \simeq 7'51897 \simeq \boxed{7'52}$ m.

3 a) Area sector ⇒ $\frac{1}{2}\theta r^2$

Area triángulo ⇒ $\frac{1}{2}r^2 \cdot \operatorname{sen}\theta$

Área región sombreada $A = \frac{1}{2}\theta r^2 - \frac{1}{2} r^2 \operatorname{sen}\theta = \boxed{\frac{1}{2}r^2(\theta - \operatorname{sen}\theta)}$

b) ratio 1:7 significa 8 partes

luego Area sombreada $\frac{1}{8}$ de la del circulo

$\frac{1}{2} r^2(\theta - \operatorname{sen}\theta) = \frac{1}{8}\pi r^2 \Rightarrow \theta - \operatorname{sen}\theta = \frac{1}{4}\pi$

Con C.G; $\boxed{\theta = 1'77}$ rad.

Si se da en º estará MAL.

Asegurarse que la calculadora esté a RAD.

Todas las operaciones a RAD, salvo que digan algo al respecto

---

4 ERROR, este problema SÍ es con C.G. (Es la P2)

a) $\hat{AC}^2 = 6^2 + 10^2 - 2 \cdot 6 \cdot 10 \cdot \cos 100°$ → poner calculadora en DEG

$AC = \boxed{12'52 \text{ cm}}$

b) $\frac{\operatorname{sen} B\hat{C}A}{6} = \frac{\operatorname{sen} 100}{12'52} \Rightarrow B\hat{C}A = \operatorname{sen}^{-1}\left(\frac{6 \times \operatorname{sen} 100}{12'52}\right) = \boxed{28'16°}$

---

5 a) $\operatorname{sen}^2 A + \cos^2 A = 1 \Rightarrow \cos^2 A = 1 - \left(\frac{5}{13}\right)^2 = \frac{169}{169} - \frac{25}{169} = \frac{144}{169}$

$\boxed{\cos A = \frac{12}{13}}$

b) $\cos 2A = \cos^2 A - \operatorname{sen}^2 A = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

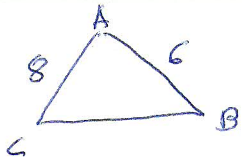
---

6 a) i) $l = \theta \cdot r = 0'7 \cdot 5 = \boxed{3'5 \text{ cm}}$

ii) perimetro $= 5 + 5 + 3'5 = \boxed{13'5 \text{ cm}}$

b) $A = \frac{1}{2}\theta \cdot r^2 = \frac{1}{2} 0'7 \cdot 5^2 = \boxed{8'75 \text{ cm}^2}$

7



a) $A = \frac{1}{2}\, 6 \cdot 8 \cdot \text{sen}\,\hat{A}$

$16 = \frac{1}{2} \cdot 6 \cdot 8 \cdot \text{sen}\,\hat{A} \Rightarrow \text{sen}\,\hat{A} = \frac{2}{3} \Rightarrow \boxed{\hat{A}_1 = 41'81^\circ}$

$\hat{A}_2 = 180 - 41'81 = \boxed{138'19^\circ}$

b) $\overline{BC}^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \cos 138'19$

$\boxed{\overline{BC} = 13'1 \text{ cm}}$

$\overline{BC} = 13'0978 \text{ cm}$

---

8 a) $l = \theta \cdot r = 1'2 \cdot 8 = \boxed{9'6}$ cm.

b) $\overline{AB}^2 = 8^2 + 8^2 - 2 \cdot 8 \cdot 8 \cdot \cos 1'2 \rightarrow \overline{AB} = 9'0342795 \approx \boxed{9'03}$ cm

c) perímetro $= 9'6 + 9'03 = \boxed{18'63}$ cm

---

9 a) $l_{ACB} = \theta \cdot r = 1'2 \cdot 10 = \boxed{12}$ cm

b) $l_{\overline{ACB}} = 2\pi r - \theta \cdot r = 50'83185 = \boxed{50'83 \text{ cm}}$

perímetro $= 10 + 10 + 50'83 = \boxed{70'83 \text{ cm}}$

---

10 a) $\sin^2 x + \cos^2 x = 1 \Rightarrow \frac{9}{16} + \cos^2 x = 1 \Rightarrow \cos x = \sqrt{1 - \frac{9}{16}} = -\frac{\sqrt{7}}{4}$

$\boxed{\cos x = -\frac{\sqrt{7}}{4}}$ obtuso

b) $\cos 2x = \cos^2 x - \text{sen}^2 x =$

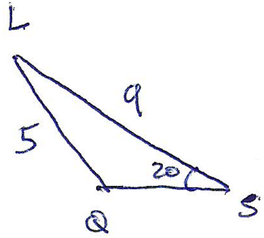
$= \frac{7}{16} - \frac{9}{16} = \frac{-2}{16} = \boxed{-\frac{1}{8}}$

---

11 a) $d = 6 \times 1'5 = 9$ Km

b) Q está + ~~de~~ cerca de Town.

i) So, Q.



ii) $\frac{\text{sen}\, 20}{5} = \frac{\text{sen}\, \hat{Q}}{9} \Rightarrow \hat{Q} = 37'9981 \approx \boxed{38^\circ}$

obtuso: $180 - 38 = \boxed{142^\circ}$

$\hat{L} = 180 - 20 - \cancel{138}\,142 = \boxed{18^\circ}$

$\frac{\text{sen}\, 18}{\overline{QS}} = \frac{\text{sen}\, 20}{5} \Rightarrow \overline{QS} = \frac{5 \cdot \text{sen}\, 18}{\text{sen}\, 20} = \boxed{4'52 \text{ Km}}$

(4)

(12)

a) $\frac{\text{sen } 35}{10} = \frac{\text{sen } 80}{\overline{AC}} \Rightarrow \overline{AC} = \frac{10 \cdot \text{sen } 80}{\text{sen } 35} = \boxed{17'17 \text{ cm.}}$

$\hat{C} = 180 - 80 - 35 = 65°$

b) $A = \frac{1}{2} \cdot 10 \cdot 17'17 \cdot \text{sen } 65° = \boxed{77'81 \text{ cm}}$

(13) teorema del coseno Δ AOB

a) $\overline{AB}^2 = r^2 + r^2 - 2 r \cdot r \cdot \cos\theta = 2r^2 - 2r^2 \cdot \cos\theta = \boxed{2r^2(1 - \cos\theta)}$

$\downarrow$

lado $\overline{AB}^2$; tb área del cuadrado.

b) Area sector: $\Rightarrow A = \frac{1}{2}\alpha \cdot r^2$

i)

ii) $\frac{1}{2}\alpha \cancel{r^2} = 2\cancel{r^2}(1 - \cos\alpha) \longrightarrow$ con C.G $\rightarrow \alpha = 0'80831 \simeq \boxed{0'81 \text{ rad}}$

OJO: RADIANES.

c) * Ver solución de este apartado al final del archivo

(14)

a) $l = \theta \cdot r = 1'3 \cdot 3 = \boxed{3'9 \text{ cm}}$

b) $A_{\widehat{ABC}} = \frac{1}{2}\theta \cdot r^2 = \frac{1}{2}(2\pi - 1'3) \cdot 3^2 = \cancel{8'29 \text{ cm}^2}\ \boxed{22'42 \text{ cm}^2}$

(15)

a) $\frac{\text{sen } 44}{\overline{AC}} = \frac{\text{sen } 83}{15} \Rightarrow \overline{AC} = \frac{15 \cdot \text{sen } 44}{\text{sen } 83} = \boxed{\cancel{0'2742}}$ cm $\boxed{10'50 \text{ cm}}$

b) $A_{ABC} = \frac{1}{2} \cdot 15 \cdot \overline{BC} \cdot \text{sen } 44 = \frac{1}{2} \cdot 15 \cdot 6'13289 \cdot \text{sen } 44 = \boxed{31'95 \text{ cm}^2}$

$\frac{\text{sen } \hat{A}}{\overline{BC}} = \frac{\text{sen } 83}{15} \Rightarrow \overline{BC} = \frac{15 \cdot \text{sen } 53°}{\text{sen } 83} = 6'13289$

$\hat{A} = 180 - 83 - 44 = 53°$

c) $A_{\Delta ACD} = \frac{1}{2} 6 \cdot 10'50 \cdot \text{sen } \theta$

$A_{\Delta ACB} = \frac{1}{2} 15 \cdot 10'50 \cdot \text{sen } 53°$

$2 \cdot \cancel{\frac{1}{2}} 6 \cdot \cancel{10'50} \cdot \text{sen }\theta = \cancel{\frac{1}{2}} 15 \cdot \cancel{10'50} \cdot \text{sen } 53$

$\theta = \boxed{86'65°}$

tb $180 - 86'65° = \boxed{93'35°}$

d) $\overline{CD}^2 = 6^2 + 10'5^2 - 2 \cdot 6 \cdot 10'5 \cdot \cos 93'35$

$\overline{CD}^2 = 153'612838 \longrightarrow \boxed{\overline{CD} = 12'39 \text{ cm}}$

16

$$A_{\Delta ABC} = \frac{1}{2} \cdot 2\sqrt{5} \cdot x \cdot \text{Sen } 2\theta \Rightarrow 5 \text{ cm}^2$$

$\text{Sen } 2\theta = 2 \cdot \text{sen } \theta \cdot \cos\theta$

$\text{Sen } 2\theta = 2 \cdot \frac{2}{3} \cdot \frac{\sqrt{5}}{3} = \frac{4\sqrt{5}}{\cancel{27}\,9}$

$\text{sen}^2\theta + \cos^2\theta = 1$

$\frac{4}{9} + \cos^2\theta = 1$

$\cos^2\theta = \frac{5}{9}$

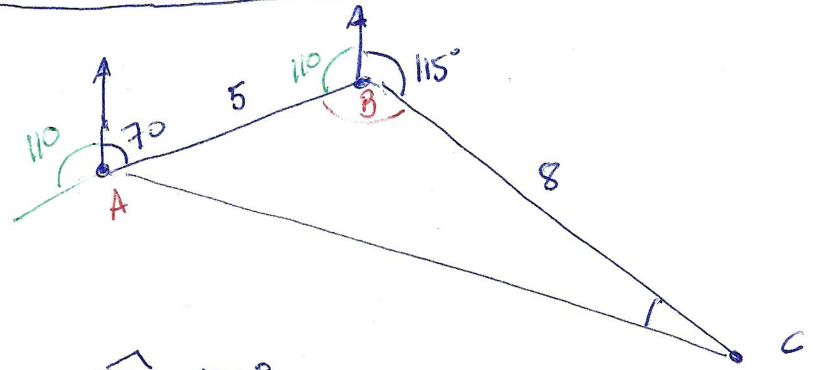
$\cos\theta = \frac{\sqrt{5}}{3}$

$$5 = \frac{1}{2} \cdot 2\sqrt{5} \cdot x \cdot \frac{4\sqrt{5}}{\cancel{3}\,9}$$

$$\frac{\cancel{5} \cdot \cancel{2} \cdot \cancel{27}\,9}{\cancel{2} \cdot 4 \cdot \cancel{5}} = x \Rightarrow \boxed{x = \frac{9\,\cancel{27}}{4} \text{ cm}} \quad \boxed{x = \frac{9}{4}}$$

17

$\widehat{ABC} = 360 - 110 - 115 = 135$



a) $\widehat{ABC} = 135°$

b) $\overline{AC}^2 = 5^2 + 8^2 - 2 \cdot 5 \cdot 8 \cdot \cos 135° \Rightarrow \overline{AC}^2 = 145'5685425$

$\boxed{\overline{AC} = 12'07 \text{ Km}}$

c) $\frac{\text{sen } \hat{C}}{5} = \frac{\text{Sen } 135}{12'07} \Rightarrow \text{sen } \hat{C} = \frac{5 \cdot \text{Sen } 135}{12'07} = 0'292919 \rightarrow \boxed{\hat{C} = 17'03°}$

18

a) $A_{\Delta ABC} = \frac{1}{\cancel{2}} \cancel{2}\sqrt{3} \cdot 6 \cdot \text{sen } \widehat{ABC}$; $3\cancel{\sqrt{3}} = \cancel{\sqrt{3}} \cdot 6 \cdot \text{sen } \widehat{ABC}$; $\frac{1}{2} = \text{sen } \widehat{ABC}$

$\widehat{ABC} = 30° \rightarrow$ Como es obtuso, $180 - 30 = \boxed{150° = \widehat{ABC}} = \boxed{\frac{5\pi}{6} \text{ rad}} = \frac{5\pi}{6}$

b) $A = \frac{1}{2} \cdot 6 \cdot \cancel{6} \cdot \frac{5\pi}{\cancel{6}} = \boxed{15\pi \text{ cm}^2}$

19 a) $\frac{\text{sen } 1'75}{\overline{BD}} = \frac{\text{sen } 0'82}{7} \Rightarrow \overline{BD} = \frac{7 \cdot \text{sen } 1'75}{\text{sen } 0'82} = 9'42$ cm

EN RADIANES

b) $12^2 = 9'42^2 + 8^2 - 9'42 \cdot 8 \cdot \cos \widehat{DBC} \rightarrow \cos \widehat{DBC} = 0'11592887$

$\boxed{\widehat{DBC} = 1'45 \text{ rad}}$

---

20 a) $\left(\frac{\sqrt{5}}{3}\right)^2 + \cos^2\theta = 1$; $\cos^2\theta = \frac{9}{9} - \frac{5}{9} = \frac{4}{9}$; $\boxed{\cos\theta = \frac{2}{3}}$

b) $\cos 2\theta = \cos^2\theta - \text{sen}^2\theta = \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2 = \frac{4}{9} - \frac{5}{9} = \boxed{-\frac{1}{9}}$

---

21 a) $\frac{2\pi}{5}$ rad $= \theta$

b) $20\cancel{\pi} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2\pi}}{5} \cdot r^2$ $\left(A = \frac{1}{2}\theta \cdot r^2\right)$

$100 = r^2$

$\boxed{10 = r}$ mm.

c) $\overline{AB}^2 = 10^2 + 10^2 - 2 \cdot 10 \cdot 10 \cdot \cos\frac{2\pi}{5} = 138'1966$. $\boxed{\overline{AB} = 11'76 \text{ mm}}$

---

22 $\frac{\text{sen } 30}{PR} = \frac{\text{sen } 45}{13} \Rightarrow \overline{PR} = \frac{13 \cdot \text{sen } 30^\circ}{\text{sen } 45^\circ} = \frac{13 \cdot \frac{1}{\cancel{2}}}{\frac{\sqrt{2}}{\cancel{2}}} = \frac{13}{\sqrt{2}} = \boxed{\frac{13\sqrt{2}}{2} \text{ cm}}$

---

23 a) $\cos\theta + 2\cos 2\theta = 1$; $\rightarrow \cos\theta + 2(2\cos^2\theta - 1) = 1$

$4\cos^2\theta + \cos\theta - 3 = 0 \rightarrow$ Ec. 2º grado.

$4x^2 + x - 3 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+48}}{8} = \frac{-1 \pm 7}{8} = \begin{cases} \boxed{x_1 = -1} \text{ No valido} \\ \boxed{x_2 = \frac{6}{8} = \frac{3}{4}} \end{cases}$ ✓

Son probabilidades

$1 \geq P(X = x_i) \geq 0$

b) $tg^2\theta + 1 = \frac{1}{\cos^2\theta}$

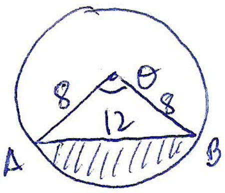
$tg^2\theta = \frac{1}{\frac{9}{16}} - 1 = \frac{16}{9} - \frac{9}{9} = \frac{7}{9} \Rightarrow \boxed{tg\,\theta = +\frac{\sqrt{7}}{3}}$

$tg\,\theta > 0$

23 c) $V = \pi \int_0^{\pi/4} \left(\frac{1}{\cos x}\right)^2 dx = \pi \cdot \left[\operatorname{tg} x \right]_0^{\pi/4} = \pi \left(\operatorname{tg}\frac{\pi}{4} - \operatorname{tg} 0 \right) =$

$= \pi \left(1 - \operatorname{tg} 0 \right) = \boxed{\pi - \frac{\pi\sqrt{7}}{3}}$

24



$12^2 = 8^2 + 8^2 - 2\cdot 8\cdot 8\cdot \cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{12^2 - 8^2 - 8^2}{(-2\cdot 8\cdot 8)}\right) = 1'696124$ rad.

$\simeq \boxed{1'70}$ rad.

$A_{segmento} = A_{sector} - A_{triángulo} = \frac{1}{2} 1'7 \cdot 8^2 - \frac{1}{2}\cdot 8^2 \cdot \operatorname{sen} 1'7 = \boxed{22'65 \text{ cm}^2}$

---

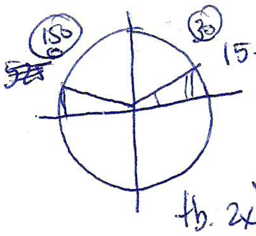
25 $\log_2 (2 \operatorname{sen} x) + \log_2 (\cos x) = -1$; $\log_2 (2 \operatorname{sen} x \cdot \cos x) = -1$; $\log_2 (\operatorname{sen} 2x) = -1$

2 posibilidades

$2^{-1} = \operatorname{sen} 2x \Rightarrow \frac{1}{2} = \operatorname{sen} 2x \Rightarrow \boxed{2x = 30}$; (1) $\boxed{x = 15°} = \boxed{\frac{\pi}{6} \text{ rad.}}$

pero, $2\pi < x < \frac{5\pi}{2}$

360° 450°



$15 + 360° = \boxed{375°}$ → solución, En radianes debe darse:

(2) tb. $2x = 150$

$\boxed{x = 75°}$

(*) $360° + 15° \Rightarrow 2\pi + \frac{\pi}{12} = \frac{24\pi + \pi}{12} = \boxed{\frac{25\pi}{12} \text{ rad.}}$

(*) $360° + 75° = 2\pi + \frac{5\pi}{12} = \frac{29\pi}{12}$ rad.

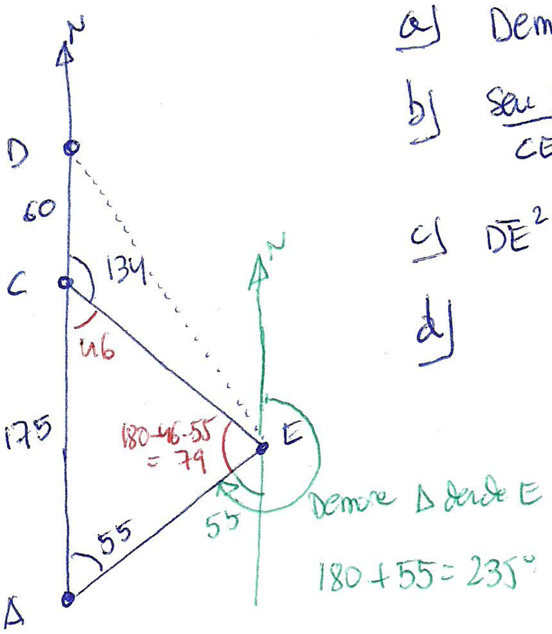
---

26 a) $l = \theta \cdot r = 1'9 \cdot 40 = \boxed{76 \text{ cm}}$

b) perímetro $= 40 + 40 + 76 = \boxed{156 \text{ cm.}}$

c) $A_{SECTOR} = \frac{1}{2} 1'9 \cdot 40^2 = \boxed{1520 \text{ cm}^2}$

27

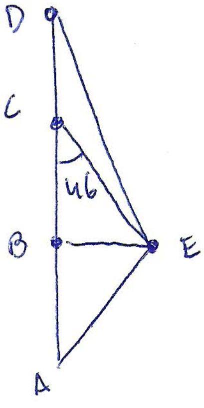


a) Demora de A desde E $= \boxed{235°}$

b) $\frac{\operatorname{sen} 55}{CE} = \frac{\operatorname{sen} 79}{175} \Rightarrow CE = \boxed{146'03}$ Km

c) $DE^2 = 60^2 + 146'03^2 - 2\cdot 60\cdot 146'03 \cdot \cos 134$; $\boxed{DE = 192'61 \text{ Km}}$

d)



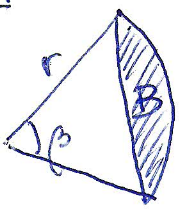
$\operatorname{sen} 46 = \frac{BE}{CE} \rightarrow \boxed{BE = 105'05 \text{ Km}}$

$V = \frac{S}{t} \rightarrow S = V \cdot t$; $t = \frac{192'61}{50} = 3'8522$ h

$V = \frac{105'05}{3'8522} = \boxed{27'27 \text{ Km/h}}$

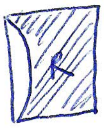
$\boxed{13\,c}$

$\boxed{13\,c}$ (*)



Área rayada = Área sector − Área triángulo =
B

$$= \frac{\beta \cdot r^2}{2} - \frac{1}{2} r^2 \cdot \operatorname{sen}\beta$$

R = 

Área de R = Área cuadrado − Área B

$$\text{Área } R = 2r^2(1-\cos\beta) - \left(\frac{\beta \cdot r^2}{2} - \frac{1}{2} r^2 \cdot \operatorname{sen}\beta\right)$$

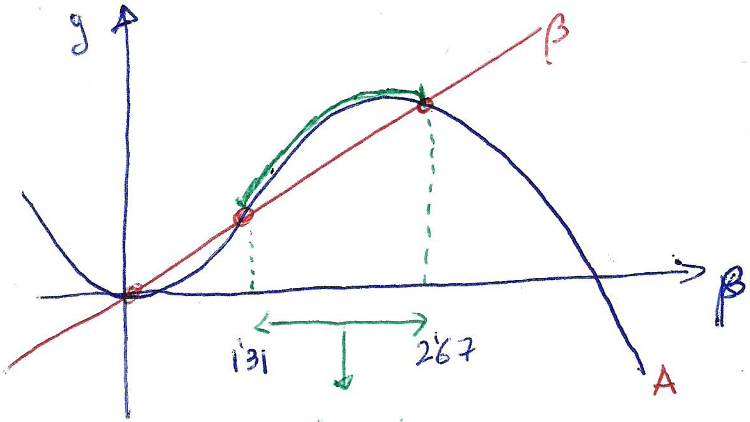
Condición R ≥ 2 Sector circular

si θ = β

$$2\cancel{r^2}(1-\cos\beta) - \left(\frac{\beta \cdot \cancel{r^2}}{2} - \frac{1}{2}\cancel{r^2} \cdot \operatorname{sen}\beta\right) \geq \cancel{2} \cdot \frac{\beta \cdot \cancel{r^2}}{\cancel{2}}$$

$$\underbrace{2(1-\cos\beta) - \left(\frac{\beta}{2} - \frac{1}{2}\operatorname{sen}\beta\right)}_{A} \geq \beta$$

Representamos los 2 lados de la inecuación en C.G. (ojo: PONER RADIANES)



intervalo en
que la ec. A
esta por encima de β

Solución

$$\beta \in [1'31, 2'67]$$