Mathematical Symbols

You will encounter many mathematical symbols during your math courses. The table below provides you with a list of the more common symbols, how to read them, and notes on their meaning and usage. The following page has a series of examples of these symbols in use.

Symbol	**How to read it**	**Notes on meaning and usage**
$a = b$	a equals b	a and b have exactly the same value.
$a \approx b$ or $a \cong b$	a is approximately equal to b	Do not write $=$ when you mean $\approx$.
$P \Rightarrow Q$	P implies Q	If P is true, then Q is also true.
$P \Leftarrow Q$	P is implied by Q	If Q is true, then P is also true.
$P \Leftrightarrow Q$ or P iff Q	P is equivalent to Q or P if and only if Q	P and Q imply each other.
(a, b)	the point a b	A coordinate in $\mathbb{R}^2$.
(a, b)	the open interval from a to b	The values between a and b, but not including the endpoints.
$[a, b]$	the closed interval from a to b	The values between a and b, including the endpoints.
$(a, b]$	The (half-open) interval from a to b excluding a, and including b.	The values between a and b, excluding a, and including b. Similar for $[a, b)$.
$\mathbb{R}$ or $\mathbf{R}$	the real numbers	It can also be used for the plane as $\mathbb{R}^2$, and in higher dimensions.
$\mathbb{C}$ or $\mathbf{C}$	the complex numbers	$\{a + bi : a, b \in \mathbb{R}\}$, where $i^2 = -1$.
$\mathbb{Z}$ or $\mathbf{Z}$	the integers	$\ldots,-2,-1,0,1,2,3,\ldots.$
$\mathbb{N}$ or $\mathbf{N}$	the natural numbers	$1, 2, 3, 4, \ldots.$
$a \in B$	a is an element of B	The variable a lies in the set (of values) B.
$a \notin B$	a is not an element of B	
$A \cup B$	A union B	The set of all points that fall in A or B.
$A \cap B$	A intersection B	The set of all points that fall in both A and B.
$A \subset B$	A is a subset of B or A is contained in B	Any element of A is also an element of B.
$\forall x$	for all x	Something is true for all (any) value of x (usually with a side condition like $\forall x > 0$).
$\exists$	there exists	Used in proofs and definitions as a shorthand.
$\exists!$	there exists a unique	Used in proofs and definitions as a shorthand.
$f \circ g$	f composed with g or f of g	Denotes $f(g(\cdot))$.
$n!$	n factorial	$n! = n(n-1)(n-2)\cdots \times 2 \times 1$.
$\lfloor x \rfloor$	the floor of x	The nearest integer $\leq x$.
$\lceil x \rceil$	the ceiling of x	The nearest integer $\geq x$.
$f = \mathcal{O}(g)$ or $f = O(g)$	f is big oh of g	$\lim_{x\to\infty} \sup_{y>x} \lvert f(y)/g(y) \rvert < \infty$. Sometimes the limit is toward 0 or another point.
$f = o(g)$	f is little oh of g	$\lim_{x\to\infty} \sup_{y>x} \lvert f(y)/g(y) \rvert = 0$.
$x \to a^+$	x goes to a from the right	x is approaching a, but x is always greater than a. Similar for $x \to a^-$.

The Trouble with =

The most commonly used, and most commonly misused, symbol is '='. The '=' symbol means that the things on either side are actually the same, just written a different way. The common misuse of '=' is to mean 'do something'. For example, when asked to compute $(3+5)/2$, some people will write:
Bad:

$$3+5 = 8/2 = 4.$$

This claims that $3+5=4$, which is false. We can fix this by carrying the '/2' along, as in $(3+5)/2 = 8/2 = 4$. We could instead use the '⇒' symbol, meaning 'implies', and turn it into a logical statement:
Good:

$$3+5=8 \quad \Rightarrow \quad (3+5)/2 = 4.$$

To Symbol or not to Symbol?

Bad: $\lim_{x\to x_0} f(x) = L$ *means that* $\forall \epsilon > 0$, $\exists \delta > 0$ *s.t.* $\forall x$,

$$0 < |x - x_0| < \delta \quad \Rightarrow \quad |f(x) - L| < \epsilon.$$

Although this statement is correct mathematically, it is difficult to read (unless you are well-versed in math-speak). This example shows that although you can write math in all symbols as a shortcut, often it is clearer to use words. A compromise is often preferred.
Good:
The Formal Definition of Limit: Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. We say that $f(x)$ approaches the limit L as x approaches x_0, and we write

$$\lim_{x\to x_0} f(x) = L$$

if for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x we have

$$0 < |x - x_0| < \delta \Longrightarrow |f(x) - L| < \epsilon.$$

Other Examples

The '⇒' symbol should be used even when doing simple algebra.
Good:

$$(y-0) = 2(x-1) \quad \Longrightarrow \quad y = 2x - 2$$

You will be more comfortable with symbols, and better able to use them, if you connect them with their spoken form and their meaning.
Good: The mathematical notation $(f \circ g)(x)$ is read "f composed with g at the point x" or "f of g of x" and means

$$f(g(x)).$$