

## Solutions to Word Problems

(involving linear equations)

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### "Age" Word Problems

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1) One-half of Heather's age two years from now plus one-third of her age three years ago is twenty years. How old is she now?

This problem refers to Heather's age two years in the future and three years in the past. So I'll pick a variable and label everything clearly:

age now:  $H$   
age two years from now:  $H + 2$   
age three years ago:  $H - 3$

Now I need certain fractions of these ages:

one-half of age two years from now:  $(\frac{1}{2})(H + 2) = \frac{H}{2} + 1$   
one-third of age three years ago:  $(\frac{1}{3})(H - 3) = \frac{H}{3} - 1$

The sum of these two numbers is twenty, so I'll add them and set this equal to 20:

$$\begin{aligned}\frac{H}{2} + 1 + \frac{H}{3} - 1 &= 20 \\ \frac{H}{2} + \frac{H}{3} &= 20 \\ 3H + 2H &= 120 \\ 5H &= 120 \\ H &= 24\end{aligned}$$

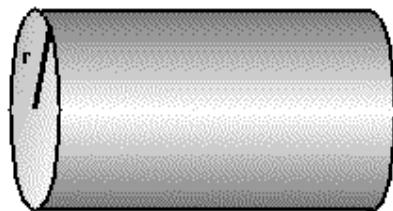
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### Geometry Word Problems

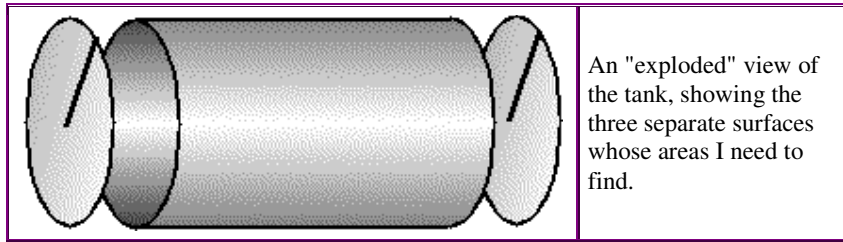
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1) Suppose a water tank in the shape of a right circular cylinder is thirty feet long and eight feet in diameter. How much sheet metal was used in its construction?

What they are asking for here is the surface area of the water tank. The total surface area of the tank will be the sum of the surface areas of the side (the cylindrical part) and of the ends. If the diameter is eight feet, then the radius is four feet. The surface area of each end is given by the area formula for a circle with radius  $r$ :  $A = (\pi)r^2$ . (There are *two* end pieces, so I will be multiplying this area by 2 when I find my total-surface-area formula.) The surface area of the cylinder is the circumference of the circle, multiplied by the height:  $A = 2(\pi)rh$ .



Side view of the cylindrical tank, showing the radius " $r$ ".



Then the total surface area of this tank is given by:

$$\begin{aligned}
 & 2 \times (\pi r^2) + 2\pi rh \quad (\text{the two ends, plus the cylinder}) \\
 & = 2(\pi (4^2)) + 2\pi (4)(30) \\
 & = 2(\pi \times 16) + 240\pi \\
 & = 32\pi + 240\pi \\
 & = 272\pi
 \end{aligned}$$

Since the original dimensions were given in terms of feet, then my area must be in terms of square feet:

**the surface area is  $272\pi$  square feet.**

**2) A piece of 16-gauge copper wire 42 cm long is bent into the shape of a rectangle whose width is twice its length. Find the dimensions of the rectangle.**

Do I care that the wire is made of copper, or that the wire is a length of sixteen-gauge? No; all I care is that the length is forty-two units, that the units are centimeters, that the rectangle is twice as long in one direction as the other, and that I'm supposed to find the values of each of these directions. I can ignore the other information.

Since the wire is 42 centimeters long, then the perimeter of the rectangle is 42 centimeters. That is:

$$2L + 2W = 42$$

I also know that the width is twice the length, so:

$$W = 2L$$

Then:

$$\begin{aligned}
 2L + 2(2L) &= 42 \quad (\text{by substitution for } W \text{ from the above equation}) \\
 2L + 4L &= 42 \\
 6L &= 42 \\
 L &= 7
 \end{aligned}$$

Since the width is related to the length by  $W = 2L$ , then  $W = 14$ , and **the rectangle is 7 centimeters long and 14 centimeters wide.**

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## "Coin" Word Problems

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1) A collection of 33 coins, consisting of nickels, dimes, and quarters, has a value of \$3.30. If there are three times as many nickels as quarters, and one-half as many dimes as nickels, how many coins of each kind are there?

I'll start by picking and defining a variable, and then I'll use [translation](#) to convert this exercise into mathematical expressions.

Nickels are defined in terms of quarters, and dimes are defined in terms of nickels, so I'll pick a variable to stand for the number of quarters, and then work from there:

$$\begin{aligned}\text{number of quarters: } & q \\ \text{number of nickels: } & 3q \\ \text{number of dimes: } & (\frac{1}{2})(3q) = (\frac{3}{2})q\end{aligned}$$

There is a total of 33 coins, so:

$$\begin{aligned}q + 3q + (\frac{3}{2})q &= 33 \\ 4q + (\frac{3}{2})q &= 33 \\ 8q + 3q &= 66 \\ 11q &= 66 \\ q &= 6\end{aligned}$$

Then **there are six quarters**, and I can work backwards to figure out that **there are 9 dimes and 18 nickels**.

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2) A wallet contains the same number of pennies, nickels, and dimes. The coins total \$1.44. How many of each type of coin does the wallet contain?

Since there is the same number of each type of coin, I can use one variable to stand for each:

$$\begin{aligned}\text{number of pennies: } & p \\ \text{number of nickels: } & p \\ \text{number of dimes: } & p\end{aligned}$$

The value of the coins is the number of cents for each coin times the number of that type of coin, so:

$$\begin{aligned}\text{value of pennies: } & 1p \\ \text{value of nickels: } & 5p \\ \text{value of dimes: } & 10p\end{aligned}$$

The total value is \$1.44, so I'll add the above, set equal to 144 cents, and solve:

$$\begin{aligned}1p + 5p + 10p &= 144 \\ 16p &= 144 \\ p &= 9\end{aligned}$$

**There are nine of each type of coin in the wallet.**

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## "Distance" Word Problems

"Distance" word problems, often also called "uniform rate" problems, involve something travelling at some fixed and steady ("uniform") pace ("rate" or "speed"), or else moving at some average speed. Whenever you read a problem that involves "how fast", "how far", or "for how long", you should think of the distance equation,  $d = rt$ , where  $d$  stands for distance,  $r$  stands for the (constant or average) rate of speed, and  $t$  stands for time.

Warning: Make sure that the units for time and distance agree with the units for the rate. For instance, if they give you a rate of feet per second, then your time must be in seconds and your distance must be in feet. Sometimes they try to trick you by using the wrong units, and you have to catch this and convert to the correct units.

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- 1) An aircraft carrier made a trip to Guam and back. The trip there took three hours and the trip back took four hours. It averaged 6 km/h on the return trip. Find the average speed of the trip there.

8 km/h

- 2) A passenger plane made a trip to Las Vegas and back. On the trip there it flew 432 mph and on the return trip it went 480 mph. How long did the trip there take if the return trip took nine hours?

10 hours

- 3) A cattle train left Miami and traveled toward New York. 14 hours later a diesel train left traveling at 45 km/h in an effort to catch up to the cattle train. After traveling for four hours the diesel train finally caught up. What was the cattle train's average speed?

10 km/h

- 4) Jose left the White House and drove toward the recycling plant at an average speed of 40 km/h. Rob left some time later driving in the same direction at an average speed of 48 km/h. After driving for five hours Rob caught up with Jose. How long did Jose drive before Rob caught up?

6 hours

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5) A passenger train leaves the train depot 2 hours after a freight train left the same depot. The freight train is traveling 20 mph slower than the passenger train. Find the rate of each train, if the passenger train overtakes the freight train in three hours.

	$d$	$r$	$t$
passenger train	$d$	$r$	3
freight train	$d$	$r - 20$	$3 + 2 = 5$
total	---	---	---

(As it turns out, I won't need the "total" row this time.) Why is the distance just " $d$ " for both trains? Partly, that's because the problem doesn't say how far the trains actually went. But mostly it's because they went the same distance as far as I'm concerned, because I'm only counting from the depot to wherever they met. After that meet, I don't care what happens. And how did I get those times? I know that the passenger train drove for three hours to catch up to the freight train; that's how I got the "3". But note that the freight train had a two-hour head start. That means that the freight train was going for five hours.

	$d$	$r$	$t$
passenger train	$d = 3r$	$r$	3
freight train	$d = 5(r - 20)$	$r - 20$	$3 + 2 = 5$
total	---	---	---

Now that I have this information, I can try to find my equation. Using the fact that  $d = rt$ , the first row gives me  $d = 3r$  (note the revised table above). The second row gives me:

$$d = 5(r - 20)$$

Since the distances are equal, I will set the equations equal:

$$3r = 5(r - 20)$$

Solve for  $r$ ; interpret the value within the context of the exercise, and state the final answer.

**6) Two cyclists start at the same time from opposite ends of a course that is 45 miles long. One cyclist is riding at 14 mph and the second cyclist is riding at 16 mph. How long after they begin will they meet?**

	$d$	$r$	$t$
slow guy	$d$	14	$t$
fast guy	$45 - d$	16	$t$
total	45	---	---

Why is  $t$  the same for both cyclists? Because I am measuring from the time they both started to the time they meet somewhere in the middle. And how did I get " $d$ " and " $45 - d$ " for the distances? Because once I'd assigned the slow guy as having covered  $d$  miles, that left  $45 - d$  miles for the fast guy to cover: the two guys *together* covered the whole 45 miles.

Using " $d = rt$ ", I get  $d = 14t$  from the first row, and  $45 - d = 16t$  from the second row. Since these distances add up to 45, I will add the distance expressions and set equal to the given total:

$$45 = 14t + 16t$$

Solve for  $t$ .

**7) A boat travels for three hours with a current of 3 mph and then returns the same distance against the current in four hours. What is the boat's speed in calm water? How far did the boat travel one way?**

	$d$	$r$	$t$
downstream	$d$	$b + 3$	3
upstream	$d$	$b - 3$	4
total	$2d$	---	7

(It may turn out that I won't need the "total" row.)

I have used " $b$ " to indicate the boat's speed. Why are the rates " $b + 3$ " and " $b - 3$ "? Because I actually have two speeds combined into one on each trip. The boat has a certain speed (the

"speed in calm water" that I'm looking for; this is the speed that registers on the speedometer), and the water has a certain speed (this is the "current"). When the boat is going with the current, the water's speed is added to the boat's speed. This makes sense, if you think about it: even if you cut the engine, the boat would still be moving, because the water would be carrying it downstream. When the boat is going against the current, the water's speed is subtracted from the boat's speed. This makes sense, too: if the water is going fast enough, the boat will *still* be going downstream (a "negative" speed, because the boat would be going backwards at this point), because the water is more powerful than the boat. (Think of a boat in a cartoon heading toward a waterfall. The guy paddles like crazy, but he still goes over the edge.)

	$d$	$r$	$t$
downstream	$d = 3(b + 3)$	$b + 3$	3
upstream	$d = 4(b - 3)$	$b - 3$	4
total	$2d$	---	7

Using " $d = rt$ ", the first row (of the revised table above) gives me:

$$d = 3(b + 3)$$

The second row gives me:

$$d = 4(b - 3)$$

Since these distances are the same, I will set them equal:

$$3(b + 3) = 4(b - 3)$$

Solve for  $b$ . Then back-solve for  $d$ .

In this case, I didn't need the "total" row.

**8) With the wind, an airplane travels 1120 miles in seven hours. Against the wind, it takes eight hours. Find the rate of the plane in still air and the velocity of the wind.**

	$d$	$r$	$t$
tailwind	1120	$p + w$	7
headwind	1120	$p - w$	8
total	2240	---	15

(I probably won't need the "total" row.) Just as with the last problem, I am really dealing with two rates together: the plane's speedometer reading, and the wind speed. When the plane turns around, the wind is no longer pushing the plane to go faster, but is instead pushing against the plane to slow it down.

The first row gives me:

$$1120 = 7(p + w)$$

The second row gives me:

$$1120 = 8(p - w)$$

The temptation is to just set these equal, like I did with the last problem, but that just gives me:

$$7(p + w) = 8(p - w)$$

...which doesn't help much. I need to get rid of one of the variables.

I'll take that first equation:

$$1120 = 7(p + w)$$

...and divide through by 7:

$$160 = p + w$$

Then, subtracting  $w$  from either side, I get that  $p = 160 - w$ . I'll substitute " $160 - w$ " for " $p$ " in the second equation:

$$1120 = 8([160 - w] - w)$$

$$1120 = 8(160 - 2w)$$

...and solve for  $w$ . Then I'll back-solve to find  $p$ .

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**9) A spike is hammered into a train rail. You are standing at the other end of the rail. You hear the sound of the hammer strike both through the air and through the rail itself. These sounds arrive at your point six seconds apart. You know that sound travels through air at 1100 feet per second and through steel at 16,500 feet per second. How far away is that spike?**

	$d$	$r$	$t$
air	$d = 1100t$	1100	$t$
steel	$d = 16,500(t - 6)$	16,500	$t - 6$
total	---	---	6

However long the sound took to travel through the air, it took six seconds less to propagate through the steel. (Since the speed through the steel is faster, then that travel-time must be shorter.) I multiply the rate by the time to get the values for the distance column. (Once again, I didn't need the "total" row.)

Since the distances are the same, I set the distance expressions equal to get:

$$1100t = 16,500(t - 6)$$

Solve for the time  $t$ , and then back-solve for the distance  $d$  by plugging  $t$  into either expression for the distance  $d$ .

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### ***"Investment" Word Problems***

Investment problems usually involve simple annual interest (as opposed to [compounded interest](#)), using the interest formula  $I = Prt$ , where  $I$  stands for the interest on the original investment,  $P$  stands for the amount of the original investment (called the "principal"),  $r$  is the interest rate (expressed in decimal form), and  $t$  is the time.

For *annual* interest, the time  $t$  must be in years. If they give you a time of, say, nine months, you must first convert this to  $\frac{9}{12} = \frac{3}{4} = 0.75$  years. Otherwise, you'll get the wrong answer. The time units must match the interest-rate units. If you got a loan from your friendly neighborhood loan shark, where the interest rate is monthly, rather than yearly, then your time must be measured in terms of months.

Investment word problems are not generally terribly realistic; in "real life", interest is pretty much always compounded somehow, and investments are not generally all for whole numbers of years. But you'll get to more "practical" stuff later; this is just warm-up, to prepare you for later.

In all cases of these problems, you will want to substitute all known information into the " $I = Prt$ " equation, and then solve for whatever is left.

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**1) You put \$1000 into an investment yielding 6% annual interest; you left the money in for two years. How much interest do you get at the end of those two years?**

In this case,  $P = \$1000$ ,  $r = 0.06$  (because I have to convert the percent to decimal form), and the time is  $t = 2$ . Substituting, I get:

$$I = (1000)(0.06)(2) = 120$$

**I will get \$120 in interest.**

Another example would be:

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**2) You invested \$500 and received \$650 after three years. What had been the interest rate?**

For this exercise, I first need to find the amount of the interest. Since interest is added to the principal, and since  $P = \$500$ , then  $I = \$650 - 500 = \$150$ . The time is  $t = 3$ . Substituting all of these values into the simple-interest formula, I get:

$$\begin{aligned} 150 &= (500)(r)(3) \\ 150 &= 1500r \\ \frac{150}{1500} &= r = 0.10 \end{aligned}$$

Of course, I need to remember to [convert](#) this decimal to a percentage.

**I was getting 10% interest.**

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The hard part comes when the exercises involve multiple investments. But there is a trick to these that makes them fairly easy to handle.

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**3) You have \$50,000 to invest, and two funds that you'd like to invest in. The You-Risk-It Fund (Fund Y) yields 14% interest. The Extra-Dull Fund (Fund X) yields 6% interest. Because of college financial-aid implications, you don't think you can afford to earn more than \$4,500 in interest income this year. How much should you put in each fund?"**

The problem here comes from the fact that I'm splitting that \$50,000 in principal into two smaller amounts. Here's how to handle this:



	$I$	$P$	$r$	$t$
Fund X	?	?	0.06	1
Fund Y	?	?	0.14	1
total	4,500	50,000	---	---

How do I fill in for those question marks? I'll start with the principal  $P$ . Let's say that I put " $x$ " dollars into Fund X, and " $y$ " dollars into Fund Y. Then  $x + y = 50,000$ . This doesn't help much, since I only know how to solve equations in one variable. But then I notice that I can solve  $x + y = 50,000$  to get  $y = 50,000 - x$ .

**THIS TECHNIQUE IS IMPORTANT!** The amount in Fund Y is (the total) less (what we've already accounted for in Fund X), or  $50,000 - x$ . You *will* need this technique, this "how much is left" construction, in the future, so make sure you understand it now.

	$I$	$P$	$r$	$t$
Fund X	?	$x$	0.06	1
Fund Y	?	$50,000 - x$	0.14	1
total	4,500	50,000	---	---

Now I will show you why I set up the table like this. By organizing the columns according to the interest formula, I can now multiply across (right to left) and fill in the "interest" column.

	$I$	$P$	$r$	$t$
Fund X	$0.06x$	$x$	0.06	1
Fund Y	$0.14(50,000 - x)$	$50,000 - x$	0.14	1
total	4,500	50,000	---	---

Since the interest from Fund X and the interest from Fund Y will add up to \$4,500, I can add down the "interest" column, and set this sum equal to the given total interest:

$$\begin{aligned}
 0.06x + 0.14(50,000 - x) &= 4,500 \\
 0.06x + 7,000 - 0.14x &= 4,500 \\
 7,000 - 0.08x &= 4,500 \\
 -0.08x &= -2,500 \\
 x &= 31,250
 \end{aligned}$$

Then  $y = 50,000 - 31,250 = 18,750$ .

**I should put \$31,250 into Fund X, and \$18,750 into Fund Y.**

Note that the answer did not involve "neat" values like "\$10,000" or "\$35,000". You should understand that this means that you cannot always expect to be able to use "guess-n-check" to find your answers. You really do need to know how to do these exercises.

If you set up your investment word problems so everything is labeled and well-organized, they should all work out fairly easily. Just take your time and do things in an orderly fashion. I've done the set-up (but not the complete solutions) for a few more examples:

**4) An investment of \$3,000 is made at an annual simple interest rate of 5%. How much additional money must be invested at an annual simple interest rate of 9% so that the total annual interest earned is 7.5% of the total investment?**

	I	P	r	t
first	$(3,000)(0.05) = 150$	3,000	0.05	1
additional	$0.09x$	$x$	0.09	1
total	$(3,000 + x)(0.075)$	$3,000 + x$	0.075	1

First I fill in the  $P$ ,  $r$ , and  $t$  columns with the given values.

Then I multiply across the rows (from the right to the left) in order to fill in the  $I$  column.

Then add down the  $I$  column to get the equation  $150 + 0.09x = (3,000 + x)(0.075)$ .

To find the solution, I would solve for the value of  $x$ .

**5) A total of \$6,000 is invested into two simple interest accounts. The annual simple interest rate on one account is 9% ; on the second account, the annual simple interest rate is 6% . How much should be invested in each account so that both accounts earn the same amount of annual interest?**

	I	P	r	t
9% account	$0.09x$	$x$	0.09	1
6% account	$(6,000 - x)(0.06)$	$6,000 - x$	0.06	1
total	---	6,000	---	---

In this problem, I don't actually need the "total" row at all.

First I'll fill in the  $P$ ,  $r$ , and  $t$  columns, and multiply to the left to fill in the  $I$  column.

From the interest column, I then get the equation  $0.09x = (6,000 - x)(0.06)$ , because the yields are required to be equal.

Then I'd solve for the value of  $x$ , and back-solve to find the value invested in the 6% account.

(This exercise's set-up used that "how much is left" construction, mentioned earlier.)

**6) An investor deposited an amount of money into a high-yield mutual fund that returns a 9% annual simple interest rate. A second deposit, \$2,500 more than the first, was placed in a certificate of deposit that returns a 5% annual simple interest rate. The total interest earned on both investments for one year was \$475. How much money was deposited in the mutual fund?**

The amount invested in the CD is defined in terms of the amount invested in the mutual fund, so I will let " $x$ " be the amount invested in the mutual fund.

	$I$	$P$	$r$	$t$
mutual fund	$0.09x$	$x$	0.09	1
cert. of deposit	$(x + 2,500)(0.05)$	$x + \$2,500$	0.05	1
total	475	$2x + \$2,500$	---	---

In this problem, I don't actually need the "total" for the "rate" or "time" columns.

First I'll fill in the  $P$ ,  $r$ , and  $t$  columns, multiplying to the left to fill in the  $I$  column.

Then I'll add down the  $I$  column to get the equation  $0.09x + (x + 2,500)(0.05) = 475$ .

Then I'd solve for the value of  $x$ .

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**7) The manager of a mutual fund placed 30% of the fund's available cash in a 6% simple interest account, 25% in 8% corporate bonds, and the remainder in a money market fund that earns 7.5% annual simple interest. The total annual interest from the investments was \$35,875. What was the total amount invested?**

For this problem, I'll let " $x$ " stand for the total amount invested.

	$I$	$P$	$r$	$t$
6% account	$(0.30x)(0.06) = 0.018x$	$0.30x$	0.06	1
8% bonds	$(0.25x)(0.08) = 0.02x$	$0.25x$	0.08	1
7.5% fund	$(0.45x)(0.075) = 0.03375x$	$0.45x$	0.075	1
total	\$35,875	$x$	---	---

Once 30% and 25% was accounted for in the 6% and 8% accounts, then there is  $100\% - 30\% - 25\% = 45\%$  left for the third account. I can use this information to fill in the "principal" column.

Then I'll fill out the "rate" and "time" columns, and multiply to the left to fill in the "interest" column.

From the interest column, I get the equation  $0.018x + 0.02x + 0.03375x = 35,875$ .

Then I'd solve for the value of  $x$ .

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## ***"Number" Word Problems***

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**1) The sum of two consecutive integers is 15. Find the numbers.**  
**"The numbers are 7 and 8."**

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**2) The product of two consecutive negative even integers is 24. Find the numbers.**

They have told me quite a bit about these two numbers: the numbers are even and they are negative. (The fact that they are negative may help if I come up with two solutions — a positive and a negative — so I'll know which one to pick.) Since even numbers are two apart (for example,  $-4$  and  $-2$  or  $10$  and  $12$ ), then I also know that the second number is two greater than the first. I also know that, when I multiply the two numbers, I will get 24. In other words, letting the first number be " $n$ " and the second number be " $n + 2$ ", I have:

$$\begin{aligned}(n)(n + 2) &= 24 \\ n^2 + 2n &= 24 \\ n^2 + 2n - 24 &= 0 \\ (n + 6)(n - 4) &= 0\end{aligned}$$

Then the solutions are  $n = -6$  and  $n = 4$ . Since the numbers I am looking for are negative, I can ignore the " $4$ " and take  $n = -6$ . Then the next number is  $n + 2 = -4$ , and the answer is

**The numbers are  $-6$  and  $-4$ .**

In the exercise above, one of the answers was one of the solutions to the equation; the other answer was the negative of the other solution to the equation. Warning: Do not assume that you can use both solutions if you just change the signs to be whatever you feel like. While this often "works", it does *not always* work, and it's sure to annoy your teacher. Throw out invalid results, and solve properly for valid ones.

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**3) Twice the larger of two numbers is three more than five times the smaller, and the sum of four times the larger and three times the smaller is 71. What are the numbers?**

The point of exercises like this is to give you practice in unwrapping and unwinding these words, and turning the words into algebraic equations. The point is in the solving, not in the relative "reality" of the problem. That said, how do you solve this? The best first step is to start labelling:

the larger number:  $x$   
the smaller number:  $y$

twice the larger:  $2x$   
three more than five times the smaller:  $5y + 3$   
relationship between ("is"):  $2x = 5y + 3$

four times the larger:  $4x$   
three times the smaller:  $3y$   
relationship between ("sum of"):  $4x + 3y = 71$

Now I have two equations in two variables:

$$\begin{aligned}2x &= 5y + 3 \\4x + 3y &= 71\end{aligned}$$

I will solve, say, the first equation for  $x$ :

$$x = (5/2)y + (3/2)$$

Then I'll plug the right-hand side of this into the second equation in place of the " $x$ ":

$$\begin{aligned}4[(5/2)y + (3/2)] + 3y &= 71 \\10y + 6 + 3y &= 71 \\13y + 6 &= 71 \\13y &= 65 \\y &= 65/13 = 5\end{aligned}$$

Now that I have the value for  $y$ , I can solve for  $x$ :

$$\begin{aligned}x &= (5/2)y + (3/2) \\x &= (5/2)(5) + (3/2) \\x &= (25/2) + (3/2) \\x &= 28/2 = 14\end{aligned}$$

As always, I need to remember to answer the question that was actually asked. The solution here is not " $x = 14$ ", but is the following sentence:

**The larger number is 14, and the smaller number is 5.**

The trick to doing this type of problem is to label everything very explicitly. Until you become used to doing these, do not attempt to keep track of things in your head. Do as I did in this last example: clearly label every single step. When you do this, these problems generally work out rather easily.

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## *"Percent of" Word Problems*

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**1) A golf shop pays its wholesaler \$40 for a certain club, and then sells it to a golfer for \$75. What is the markup rate?**

First, I'll calculate the markup in absolute terms:

$$75 - 40 = 35$$

Then I'll find the relative markup over the original price, or the markup rate: (\$35) is (some percent) of (\$40), or:

$$35 = (x)(40)$$

...so the relative markup over the original price is:

$$35 \div 40 = x = 0.875$$

Since  $x$  stands for a percentage, I need to remember to convert this decimal value to the corresponding percentage.

**The markup rate is 87.5%.**

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**2) A shoe store uses a 40% markup on cost. Find the cost of a pair of shoes that sells for \$63.**

This problem is somewhat backwards. They gave me the selling price, which is cost plus markup, and they gave me the markup rate, but they didn't tell me the actual cost or markup. So I have to be clever to solve this.

I will let " $x$ " be the cost. Then the markup, being 40% of the cost, is  $0.40x$ . And the selling price of \$63 is the sum of the cost and markup, so:

$$\begin{aligned} 63 &= x + 0.40x \\ 63 &= 1x + 0.40x \\ 63 &= 1.40x \\ 63 \div 1.40 &= x = 45 \end{aligned}$$

**The shoes cost the store \$45.**

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**3) An item originally priced at \$55 is marked 25% off. What is the sale price?**

First, I'll find the markdown. The markdown is 25% of the original price of \$55, so:

$$x = (0.25)(55) = 13.75$$

By subtracting this markdown from the original price, I can find the sale price:

$$55 - 13.75 = 41.25$$

**The sale price is \$41.25.**

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**4) An item that regularly sells for \$425 is marked down to \$318.75. What is the discount rate?**

First, I'll find the amount of the markdown:

$$425 - 318.75 = 106.25$$

Then I'll calculate "the markdown over the original price", or the markdown rate: (\$106.25) is (some percent) of (\$425), so:

$$106.25 = (x)(425)$$

...and the relative markdown over the original price is:

$$x = 106.25 \div 425 = 0.25$$

Since the "x" stands for a percentage, I need to remember to convert this decimal to percentage form.

**The markdown rate is 25%.**

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**5) An item is marked down 15%; the sale price is \$127.46. What was the original price?**

This problem is backwards. They gave me the sale price (\$127.46) and the markdown rate(15%), but neither the markdown amount nor the original price. I will let "x" stand for the original price. Then the markdown, being 15% of this price, was 0.15x. And the sale price is the original price, less the markdown, so I get:

$$\begin{aligned}x - 0.15x &= 127.46 \\1x - 0.15x &= 127.46 \\0.85x &= 127.46 \\x &= 127.46 \div 0.85 = 149.952941176...\end{aligned}$$

This problem didn't state how to round the final answer, but dollars-and-cents is always written with two decimal places, so:

**The original price was \$149.95.**

Note in this last problem that I ended up, in the third line of calculations, with an equation that said "eighty-five percent of the original price is \$127.46". You can save yourself some time if you think of discounts in this way: if the price is 15% off, then you're only actually paying 85%. Similarly, if the price is 25% off, then you're paying 75%; if the price is 30% off, then you're paying 70%; and so on.

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## Mixture Word Problems

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- 1) 2 m<sup>3</sup> of soil containing 35% sand was mixed into 6 m<sup>3</sup> of soil containing 15% sand. What is the sand content of the mixture?

20%

- 2) 9 lbs. of mixed nuts containing 55% peanuts were mixed with 6 lbs. of another kind of mixed nuts that contain 40% peanuts. What percent of the new mixture is peanuts?

49%

- 3) 5 fl. oz. of a 2% alcohol solution was mixed with 11 fl. oz. of a 66% alcohol solution. Find the concentration of the new mixture.

46%

- 4) 16 lb of Brand M Cinnamon was made by combining 12 lb of Indonesian cinnamon which costs \$19/lb with 4 lb of Thai cinnamon which costs \$11/lb. Find the cost per lb of the mixture.

\$17/lb

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## Work Word Problems

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- 1) Working alone, Ryan can dig a 10 ft by 10 ft hole in five hours. Castel can dig the same hole in six hours. How long would it take them if they worked together?

2.73 hours

- 2) Shawna can pour a large concrete driveway in six hours. Dan can pour the same driveway in seven hours. Find how long it would take them if they worked together.

3.23 hours

- 3) It takes Trevon ten hours to clean an attic. Cody can clean the same attic in seven hours. Find how long it would take them if they worked together.

4.12 hours

- 4) Working alone, Carlos can oil the lanes in a bowling alley in five hours. Jenny can oil the same lanes in nine hours. If they worked together how long would it take them?

3.21 hours

- 5) Working together, Paul and Daniel can pick forty bushels of apples in 4.95 hours. Had he done it alone it would have taken Daniel 9 hours. Find how long it would take Paul to do it alone.

11 hours

- 6) Working together, Jenny and Natalie can mop a warehouse in 5.14 hours. Had she done it alone it would have taken Natalie 12 hours. How long would it take Jenny to do it alone?

8.99 hours